The Proximity Effect: A Comparison of COMSOL® and Analytic Solutions

(A Possible Means for Accuracy Validation ?)

Chathan Cooke[1], Lisa Shatz[2,3]

1) MIT: RLE, (High Voltage Lab.), Cambridge, MA, USA

2) Suffolk University Electrical Engineering, Boston, MA, USA

3) Benjamin Franklin Institute of Technology, Boston, MA, USA

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Newton, MA

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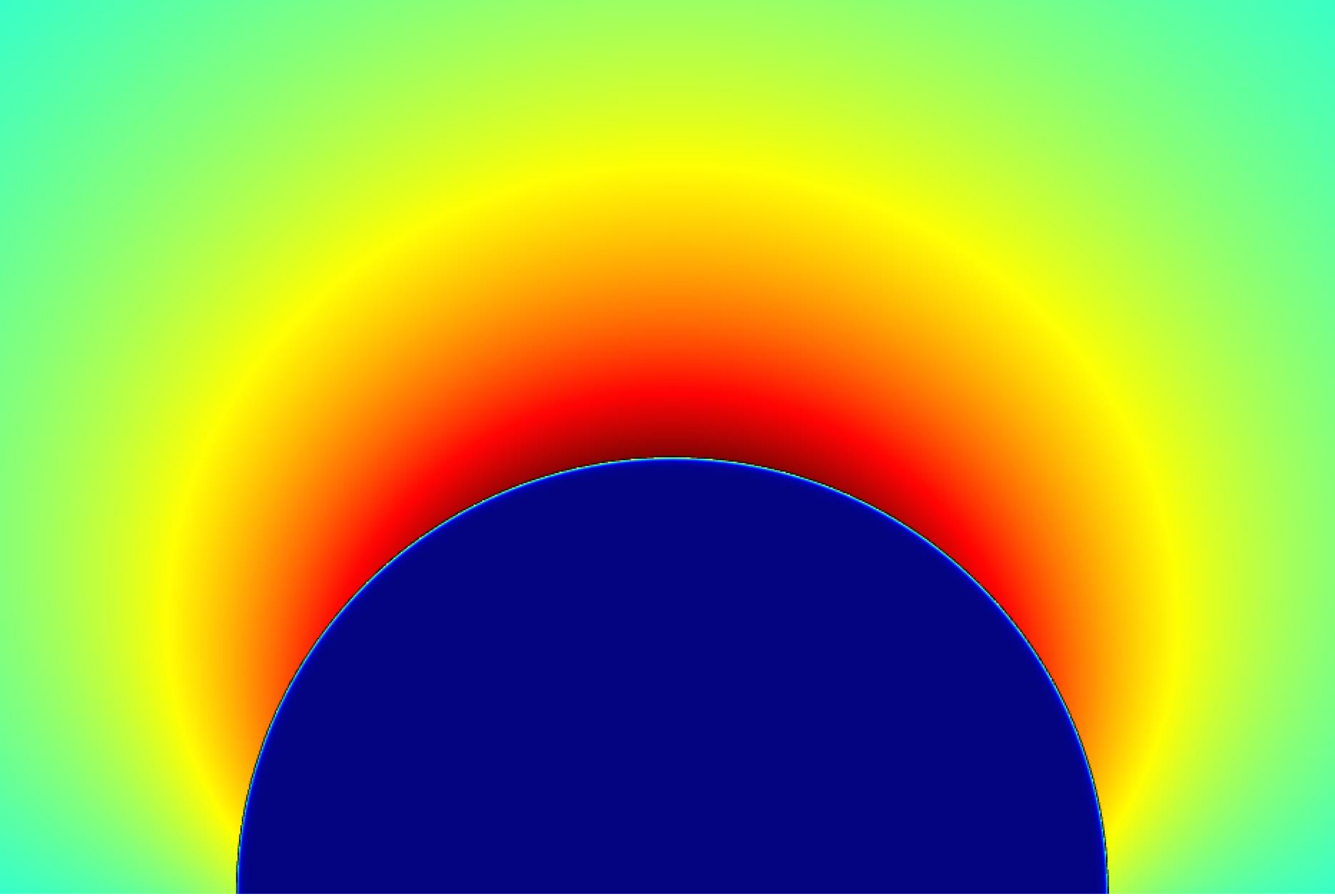

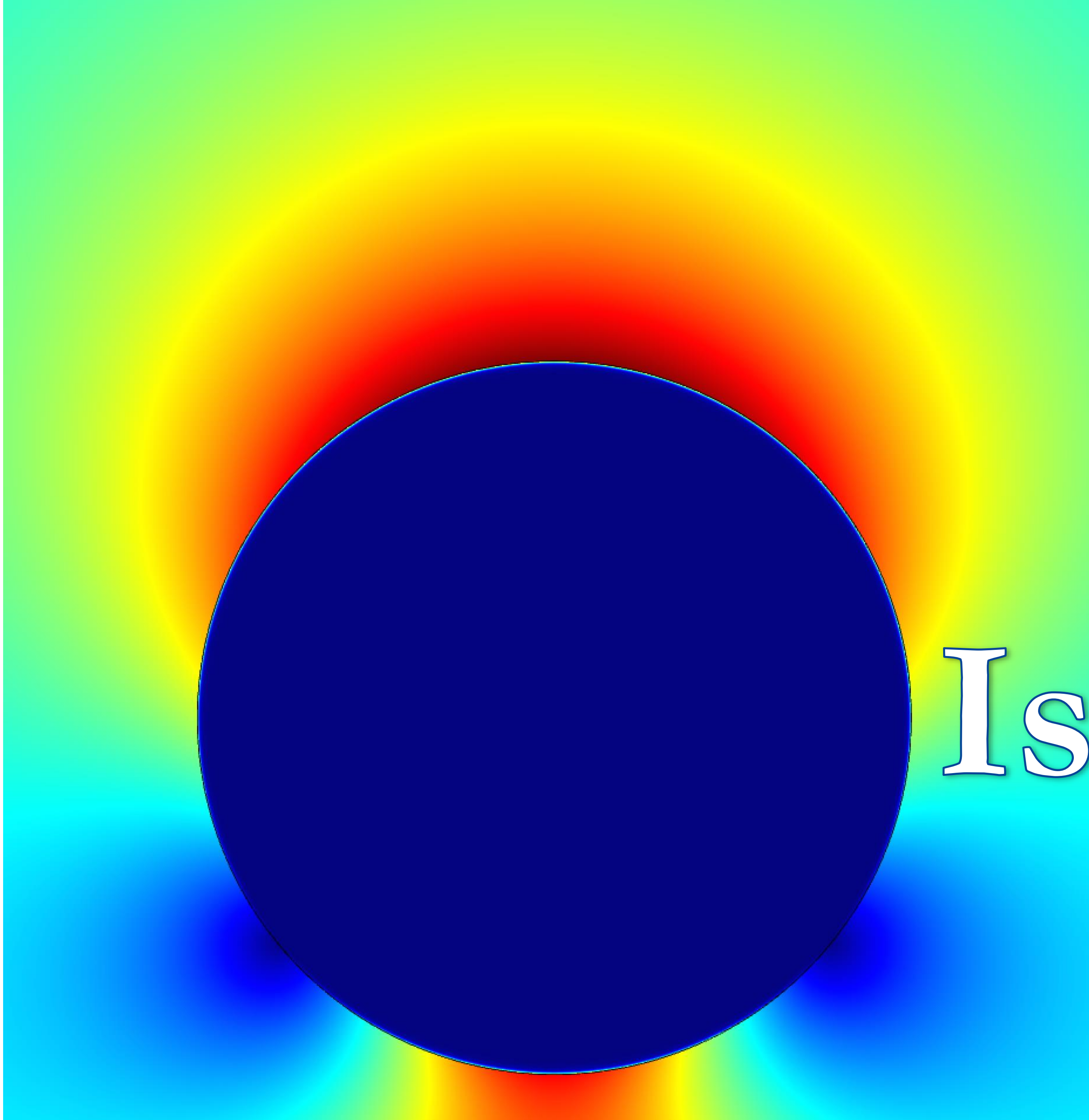
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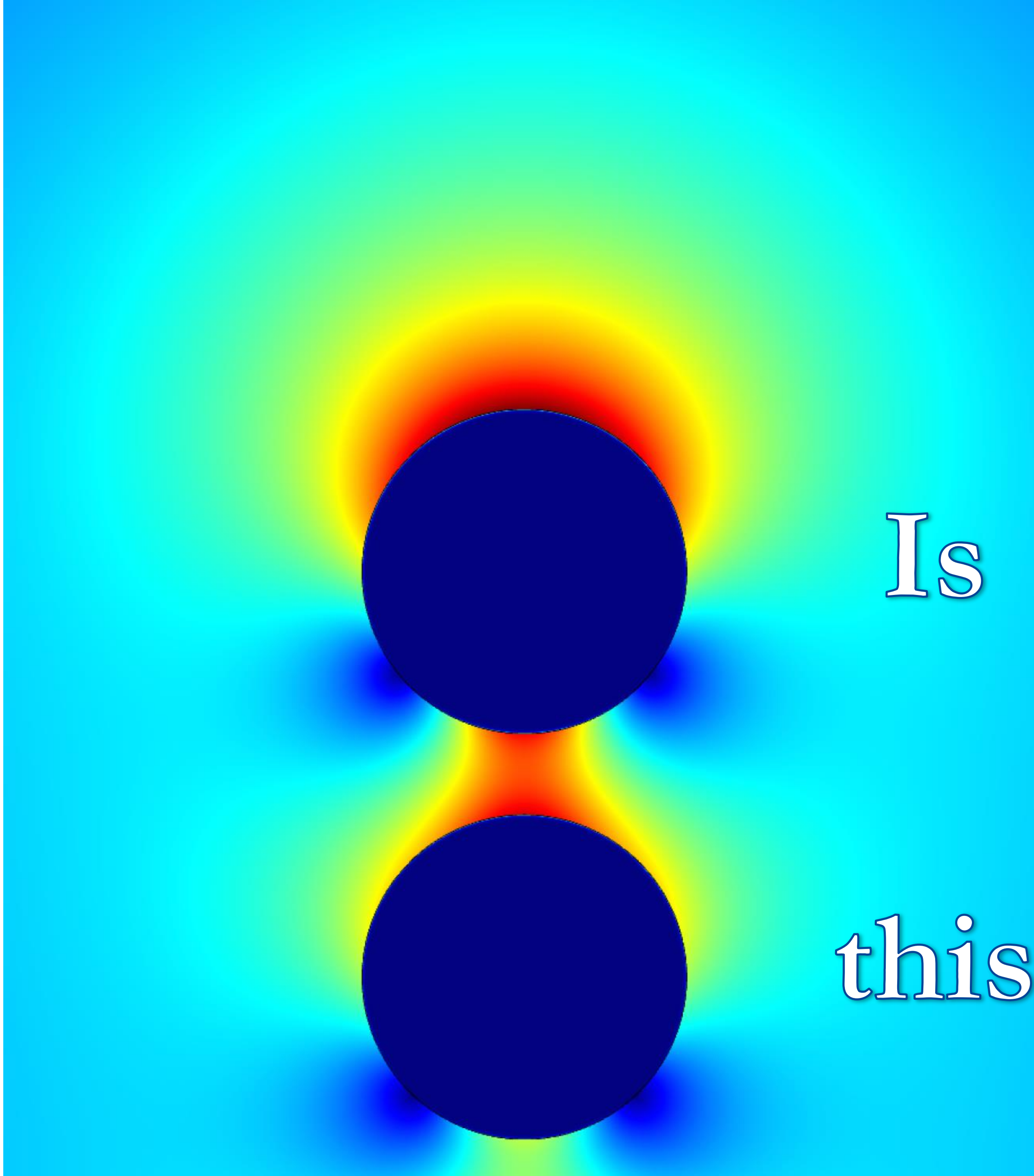
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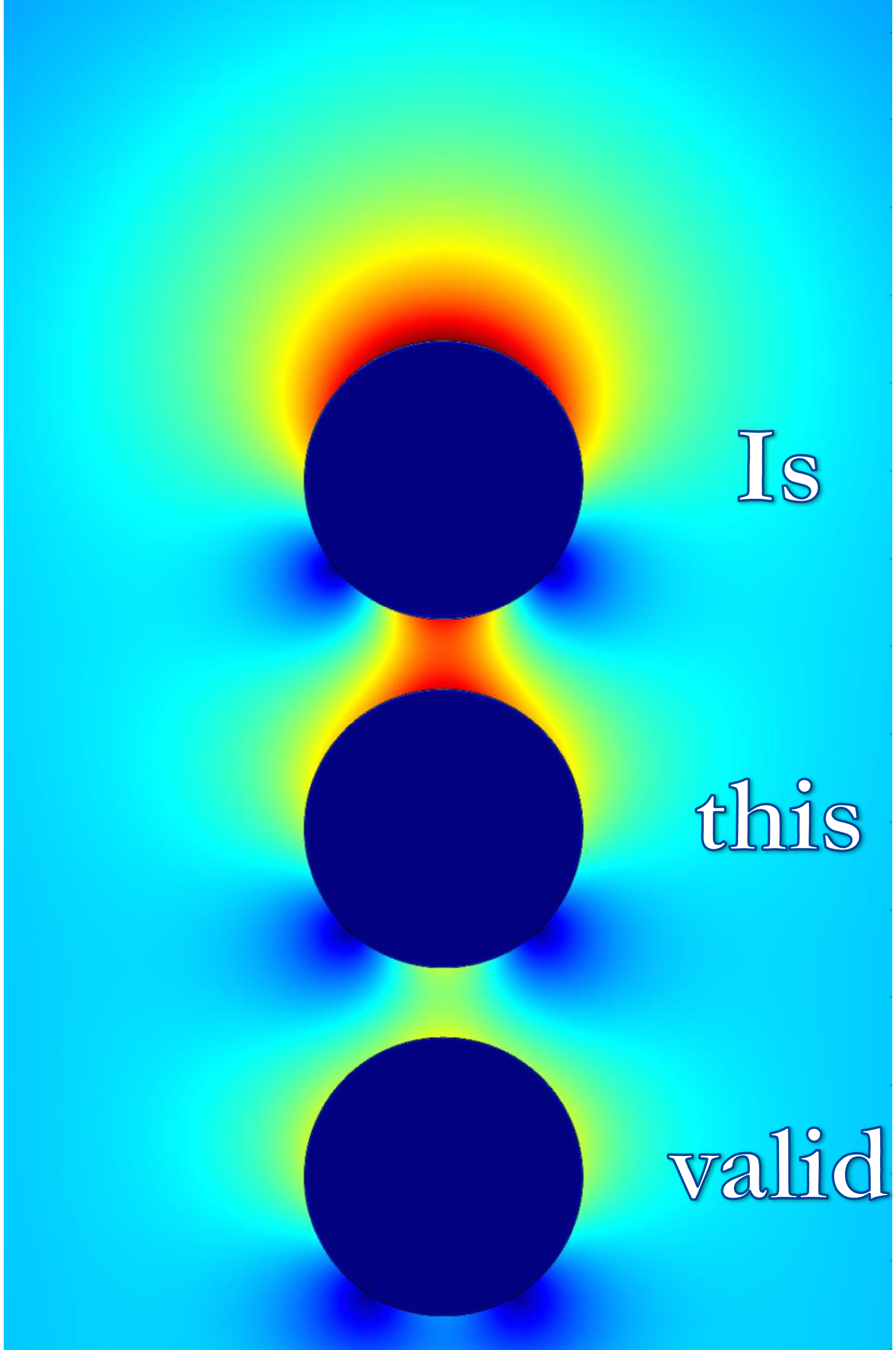
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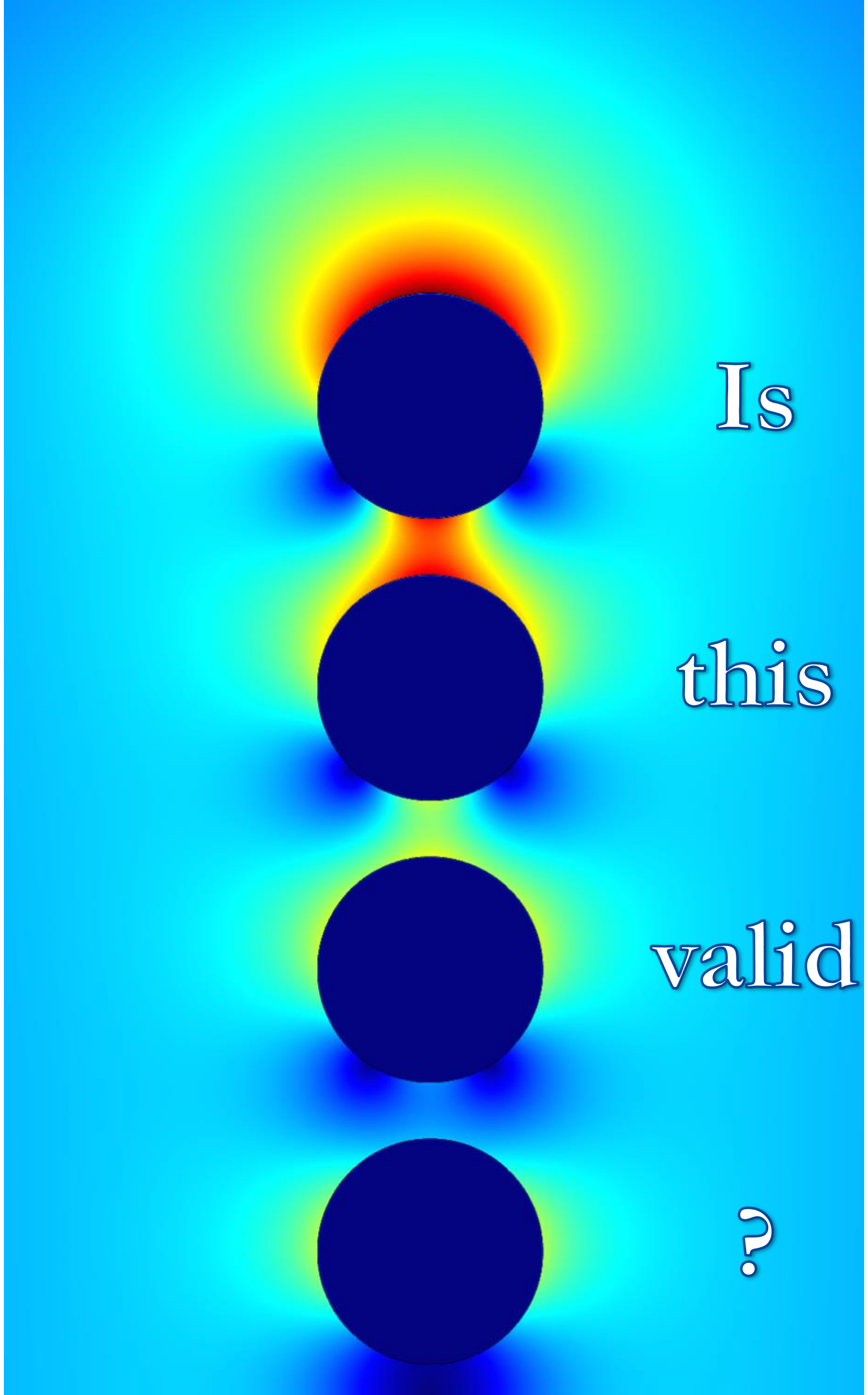
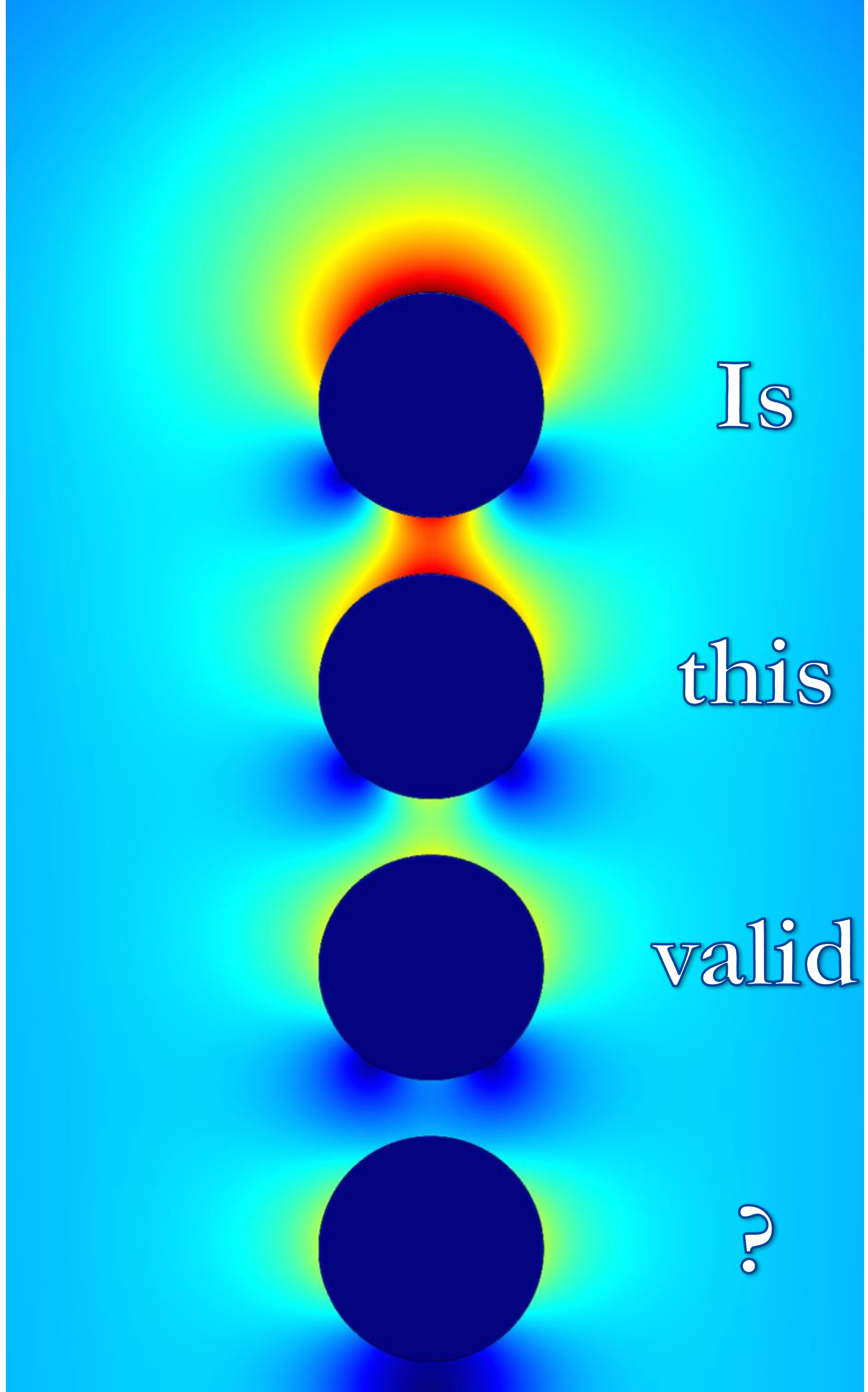

Is
this
valid
?

8 Parallel Wire
Magnetic
Flux Density

with

Equal
Total Current
in Each Wire



Non-Equal
Current
Densities

the

Cause:
Proximity
Effect

COMSOL2019

Outline of Topics

1) Introduction / Goals
   – Desire: challenging problem with analytic (exact) `solution

2) Selected Problem for "Validation"
   – Has analytic solution for point and volume values

3) Mathematica® (analytic) and COMSOL® (simulation)
   – Obtain quantified values
   – Compare to analytic solution for accuracy

4) Conclusions

Goals of "Validation" Problem

Analytic (Exact) vs Simulation

1) Compatible with Modern Solvers:
   - Requires Only Common, Expected Capability
     - No "Special Physics"

2) Challenging Problem, "Not Simple"
   - Very high dynamic range of field values
     - Severe challenge to numerical algorithms
   - 2D: Both radial and angular variations

3) Challenging Post-Processing
   - For example: bulk electrical properties
     - Quantified resistance, added power-loss

Selected: Proximity Effect

2D - Quasi-Static EM Problem

Demanding for all three above goals

Proximity Effect:

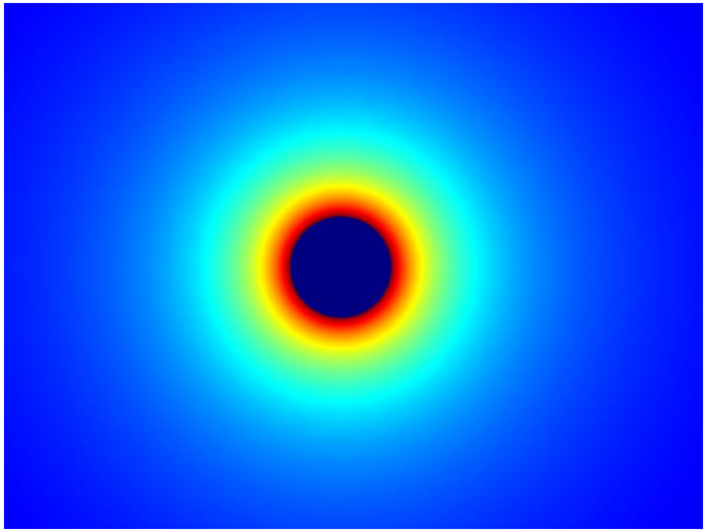
"External" Currents Influence "Internal" Currents

Induces a redistribution of currents

Single Wire

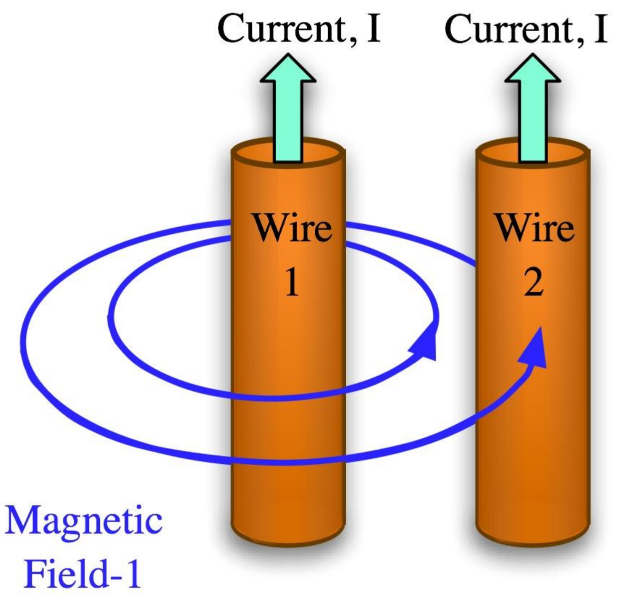
End View

Uniform Distribution

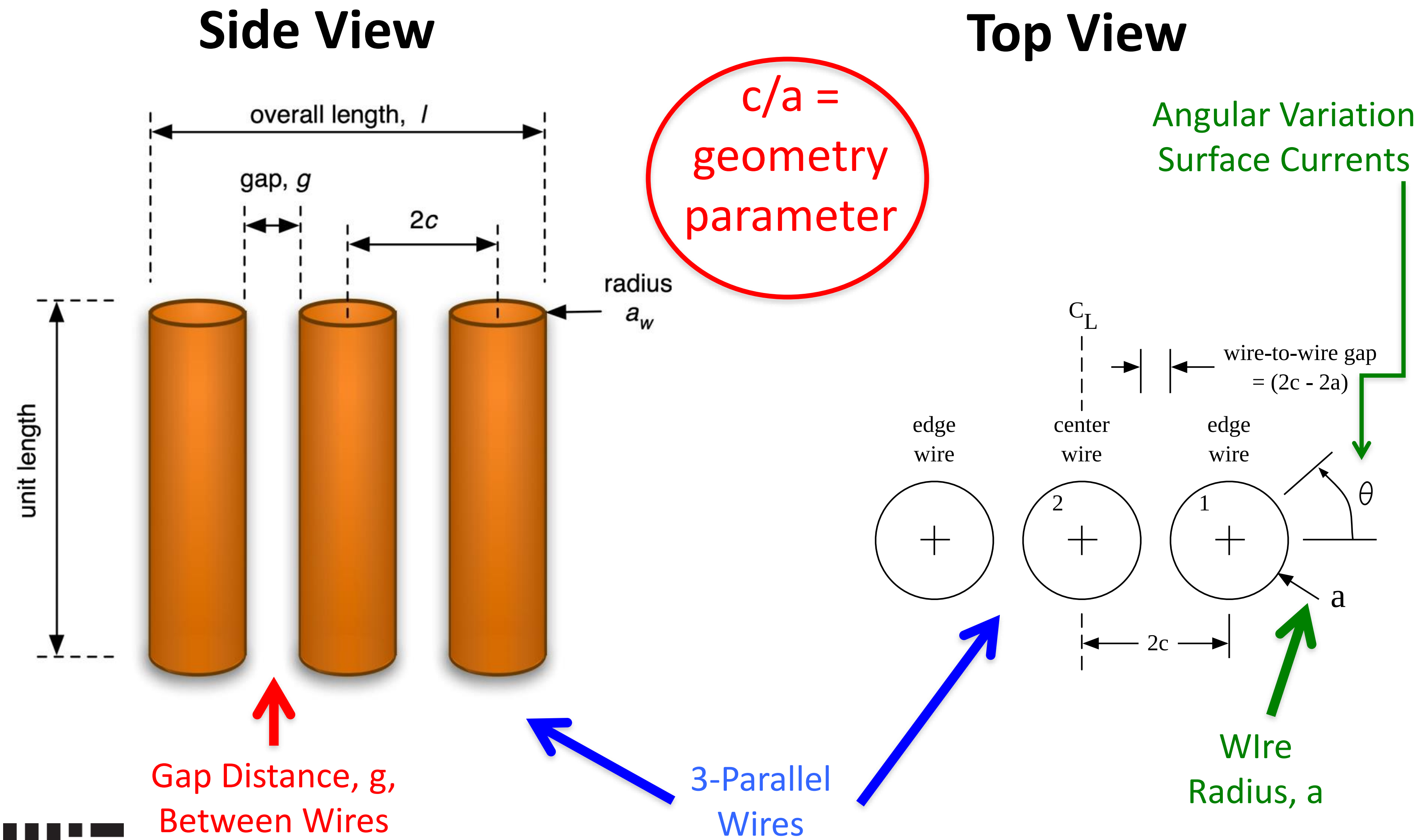


Multiple Wires

Non-Uniform Distribution



Parallel Wire Geometry: 3 Wires



Analytic Calculation Method

(After Smith 1972)

Assumptions, Definitions:

1) All wires carry same total current, *I*:

angular variation of surface current density

2) ONLY Surface Current Density, *K*, with angular variation g_m:

$$K_l(\theta', z') = \left(I \big/ 2\pi a \right) g_l(\theta') \qquad \text{(A/m)}$$

at m^th wire: coefficients, a_{mp}

3) By symmetry only cosine terms for: $g_m(\theta)$

$$g_m(\theta) = g_m(\pi - \theta)$$

$$g_m(\theta') = 1 + \sum_{p=1}^{\infty} a_{mp} \cos(p\theta')$$

4) Magnetic vector potential is z-directed:

$$\mathbf{A} = A_{mz}(r, \theta, z)\hat{\mathbf{z}}$$

$$-\frac{\partial (A_{mz}(r, \theta, z))}{\partial r} = \left(\mu_o I \big/ 2\pi a \right) g_l(\theta)$$

Analytic Form for Surface Current Distributions

(Series of Integral Equations for $g_m(\theta)$)

Integral Equations

$$g_m(\theta) = 1 + \frac{1}{\pi} \int_{-\pi}^{\pi} \left(\sum_{l=1, l \neq m}^{m} g_l(\theta') K_{ml}(\theta, \theta') \, d\theta' \right)$$

Where:

$$K_{ml}(\theta, \theta') = \frac{1 + 2(c/a)(m - l)\cos\theta - \cos(\theta - \theta')}{(r'_{ml})^2},$$

Angle Dependent Coefficients

normalized by wire-spacing: (c/a)

Hence, Result:

$$g_m(\theta) = 1 + \sum_{p=1}^{q} a_{mp} \cos(p\theta)$$

(Calculate $g_m(\theta)$ via Series Coefficients: a_{mp})

Where: summation to q = number of cosine terms to get convergence
(typically 6 to 8)

Example Analytic Calculation

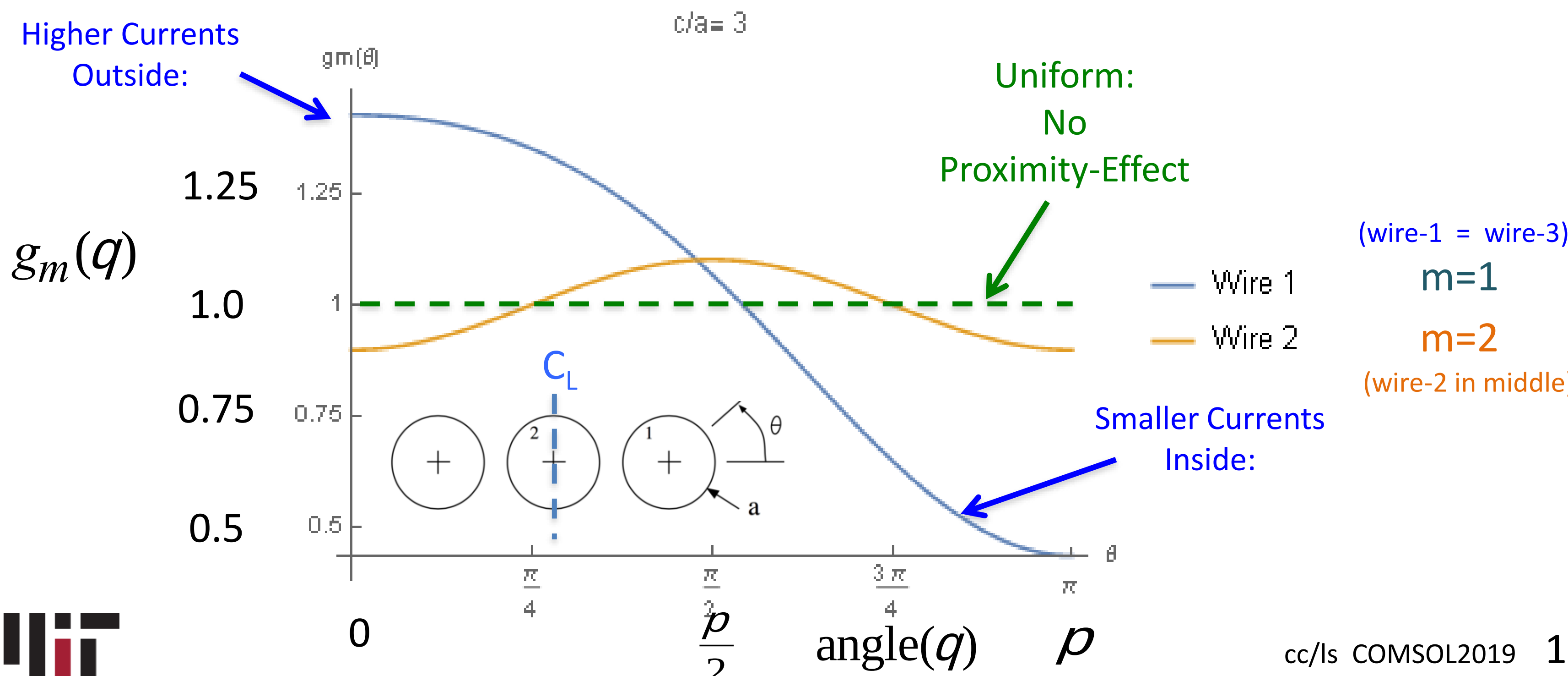
(Three conductors: n=3, Two terms: q = 2)

$$g_m(q) = 1 + a_{m1}\cos q + a_{m2}\cos 2q$$

For wire *m*

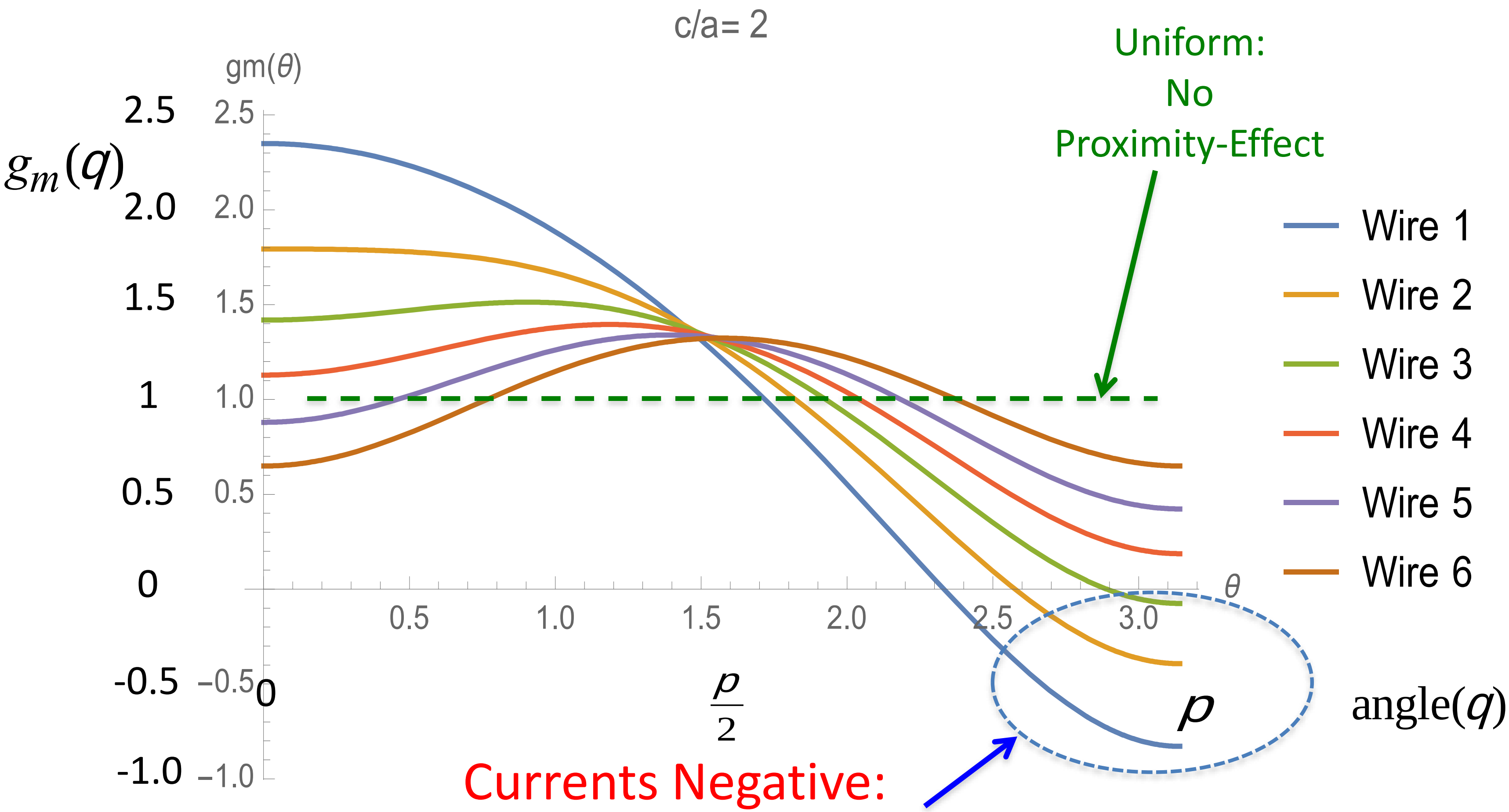
Solve for the case of c/a = 3, and get:

$$a_{11} = .496,\ a_{12} = .069,\ a_{21} = 0,\ a_{22} = .102$$



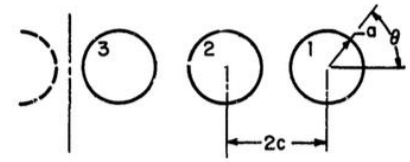
Example Analytic Solution

11-Wires, (c/a) = 2

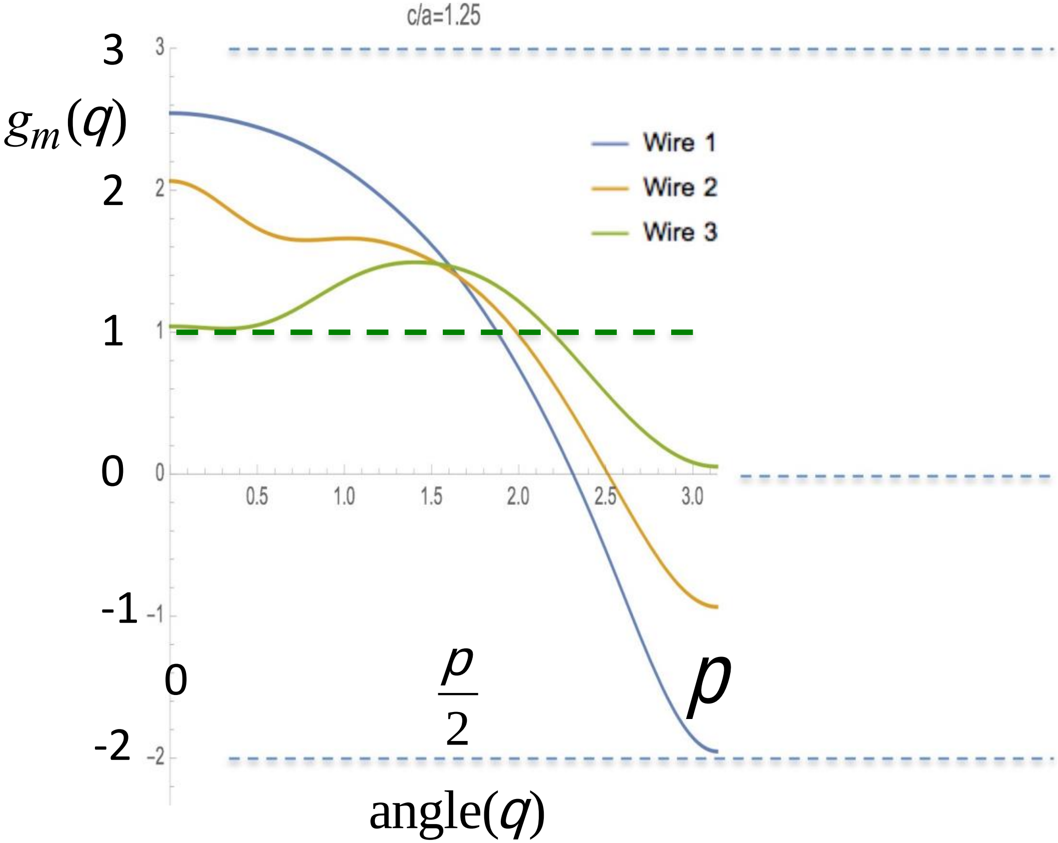


Compare Analytic Solution, Smith (1972)

6-Wires, (c/a) = 1.25



Analytic via Mathematica®



Overlay on Smith (1972)

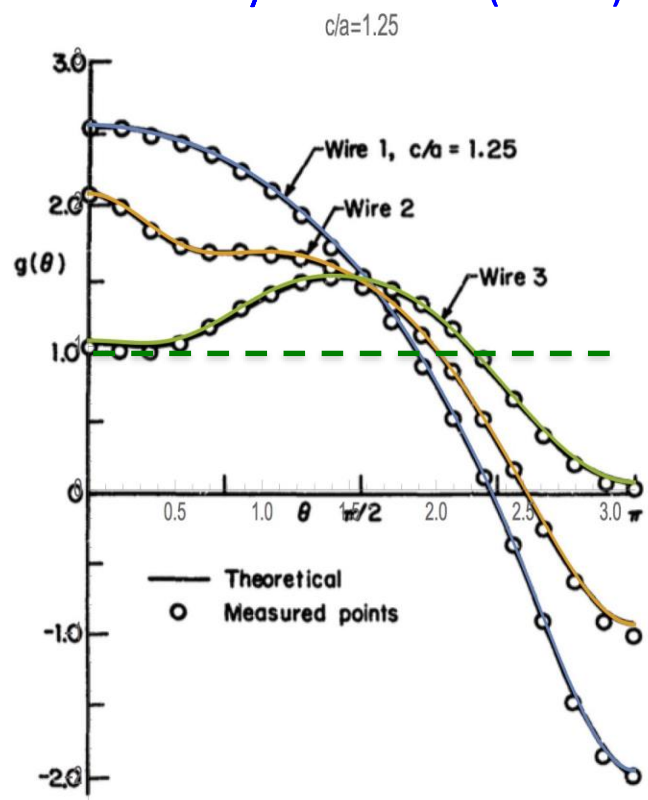


FIG. 3-11 MEASURED AND THEORETICAL SURFACE CURRENT DISTRIBUTIONS FOR SIX WIRES

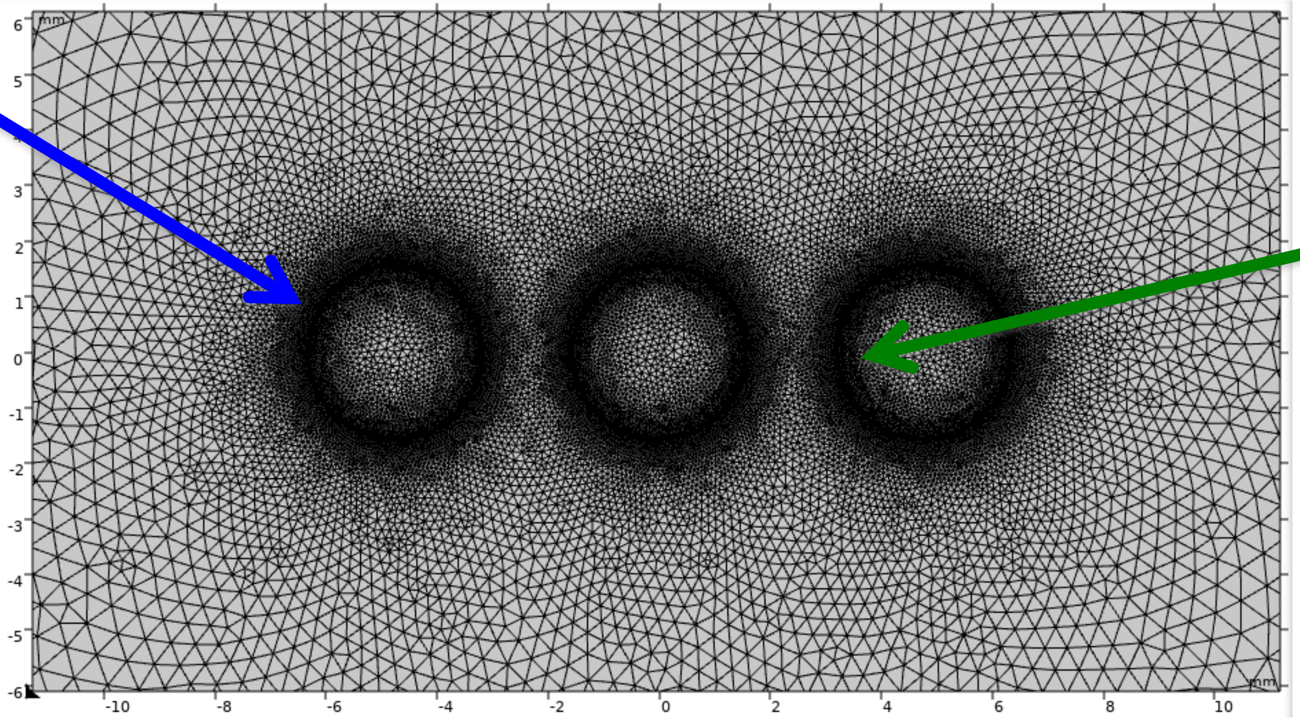
Solver Simulation Solutions

Via COMSOL ® with AC/DC Module

Solver Simulation Method - Meshing
(COMSOL® + AC/DC Module)

Meshing: 3 parallel wires

high
mesh-density
Just outside
wire surface



high
mesh-density
just inside
wire surface

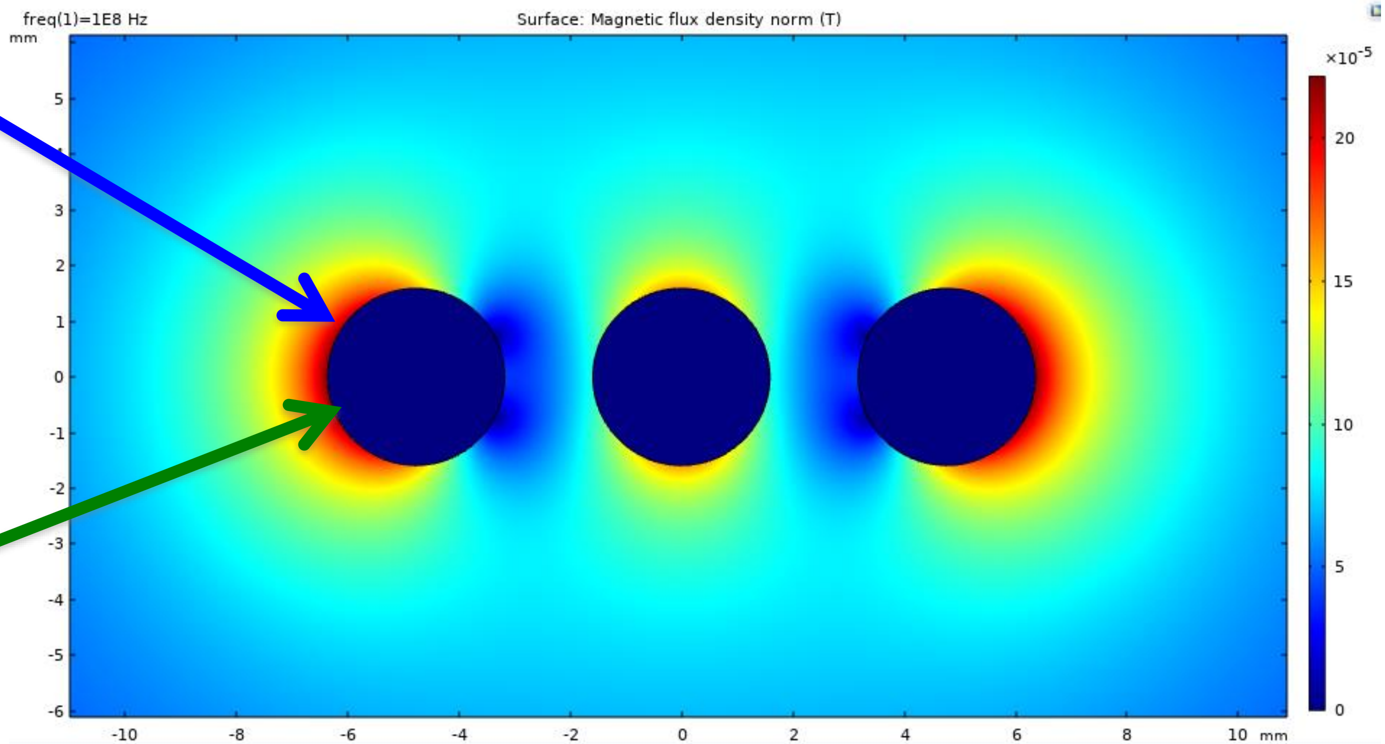
"infinite" boundary

Solver Simulation Method – Magnetic Flux Density
(COMSOL® + AC/DC Module)

3 wires, (c/a)= 1.5, skin depth = 6.3µm (freq = 100MHz)

high fields outside wire surface

high current density Inside wire surface



1 amp Each wire

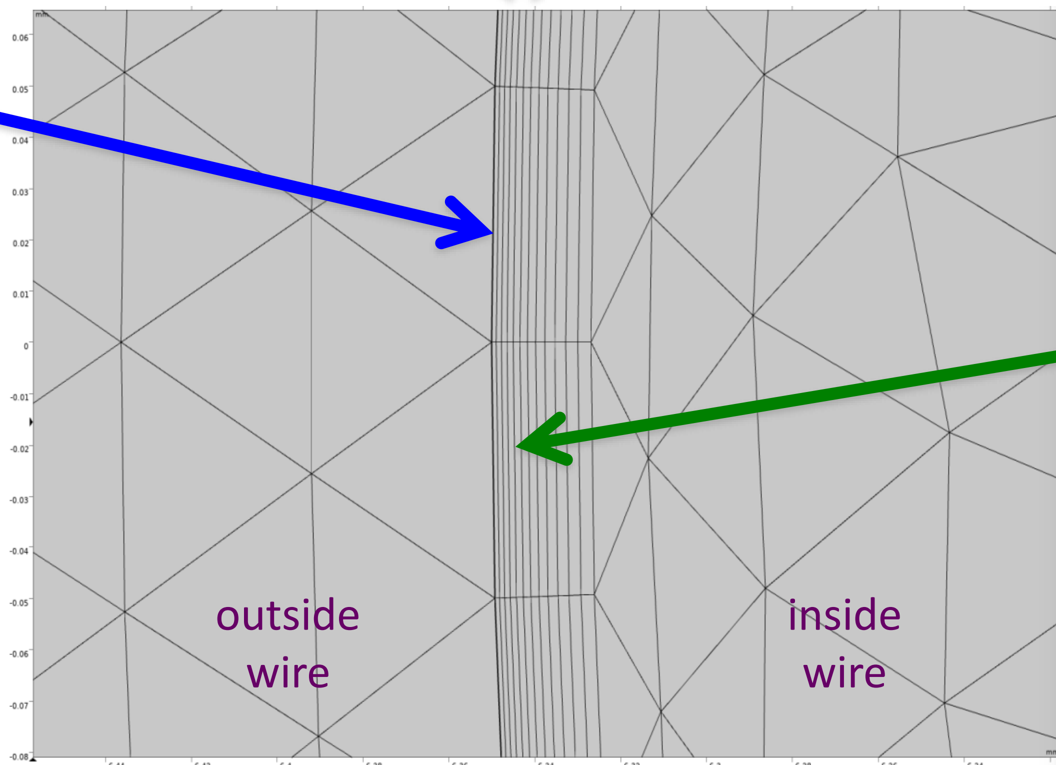
"infinite" boundary

Expanded View - Meshing
(COMSOL® + AC/DC Module)

Ultra Fine Meshing: 3 wires (c/a)= 1.5

10 µm

Wire surface



high mesh density
Just Inside
wire surface

skin depth
= ~6.3 µm

(5 mesh layers)

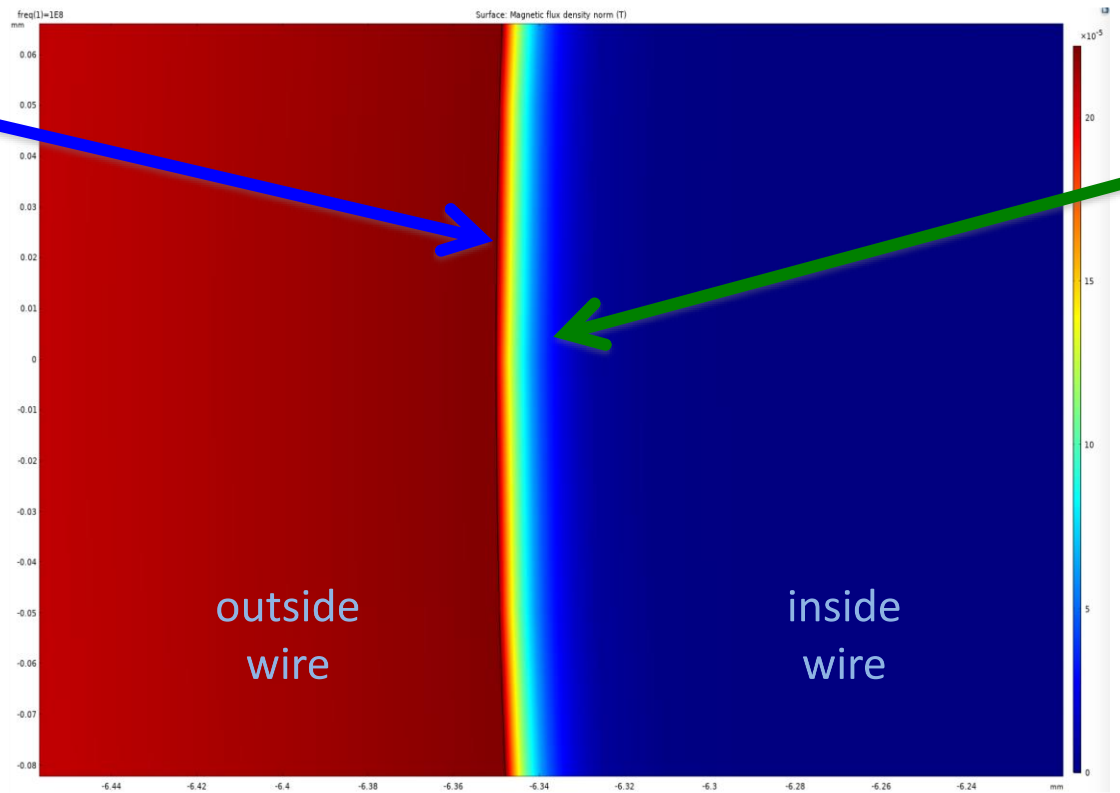
200 µm

Expanded View – Magnetic Flux Density
(COMSOL® + AC/DC Module)

3 wires, (c/a) = 1.5

10 µm

Wire surface



high current density Inside wire surface

skin depth = ~6.3 µm

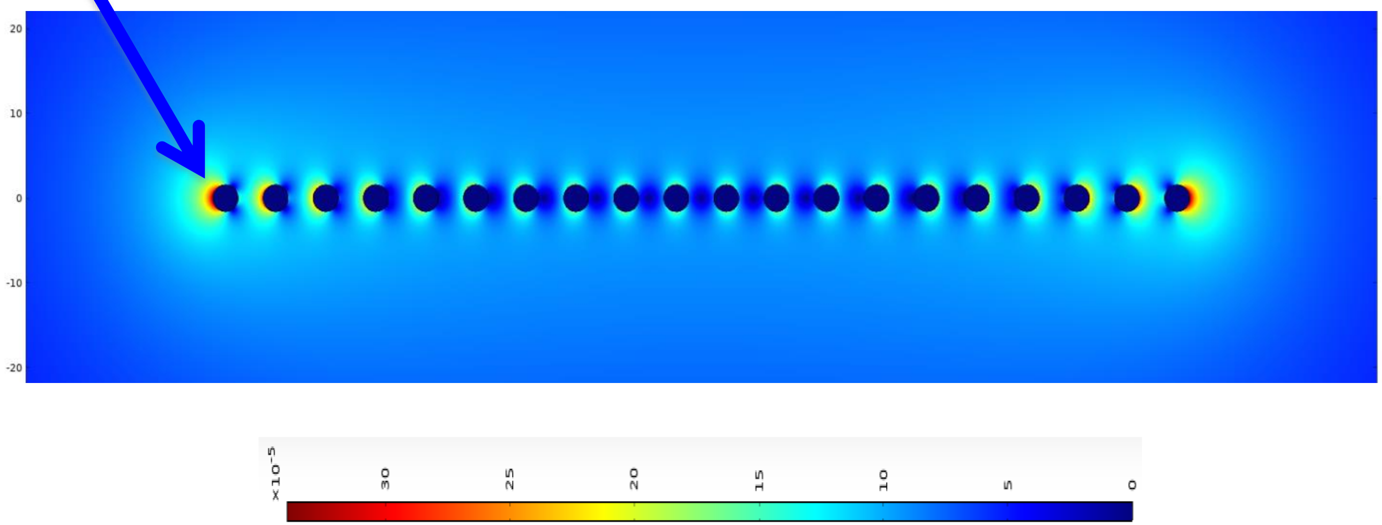
200 µm

Solver Simulation Method – Magnetic Flux Density
(COMSOL® + AC/DC Module)

20 wires, (c/a) = 2.0, freq = 100MHz (skin depth = 6.3μm)

higher fields
outside
outer wire surface

All wires
1 amp current

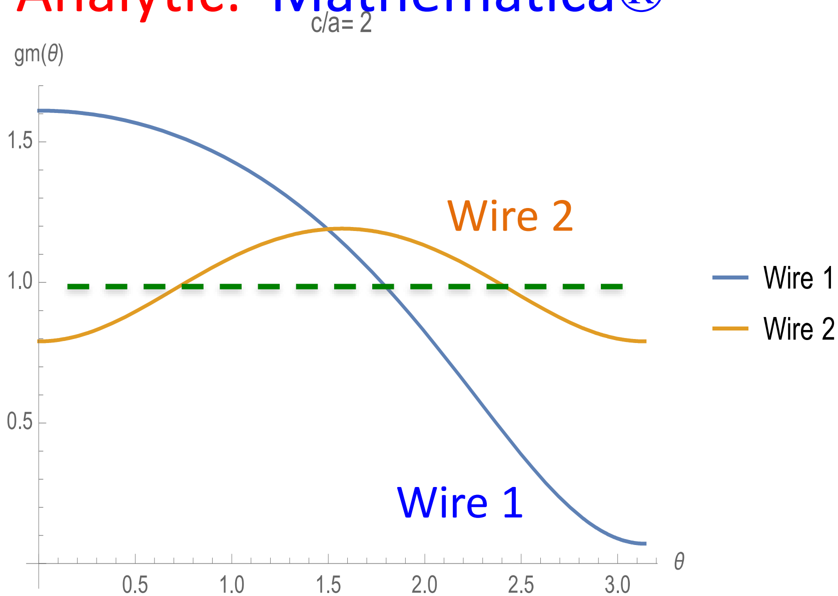


35 30 20 10 0 (x 10^{-5} Tesla)

Comparison: Analytic vs COMSOL®

3-Wires, (c/a) = 2: Surface Current Density Distributions

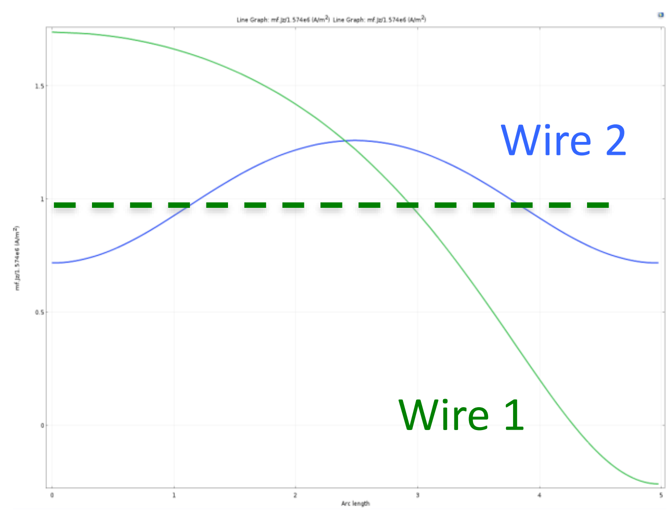
Analytic: Mathematica®



0 $\frac{p}{2}$ p

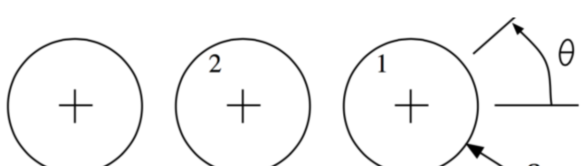
Angle

Simulation: COMSOL®



0 $\frac{p}{2}$ p

Angle



Proximity Effect:
2nd Calculation Quantity = Resistance
(a bulk volume property)

Added Resistance per Wire:

R_p/R_o

(normalized to skin-effect R_o)

Calculation of Normalized Proximity Resistance: Rp/Ro

- Analytic: Use same a_{mp} coefficients for $g_m(q)$:

$$\frac{R_p}{R_o} = \frac{R_T - NR_{skin}}{NR_{skin}} = \frac{1}{2}\sum_{m=1}^{N}\left(\sum_{p=1}^{g}\left|a_{mp}\right|^2\right)$$

- Simulation: COMSOL® post-process, via:

"Volumetric loss density, electric" function [W/m]

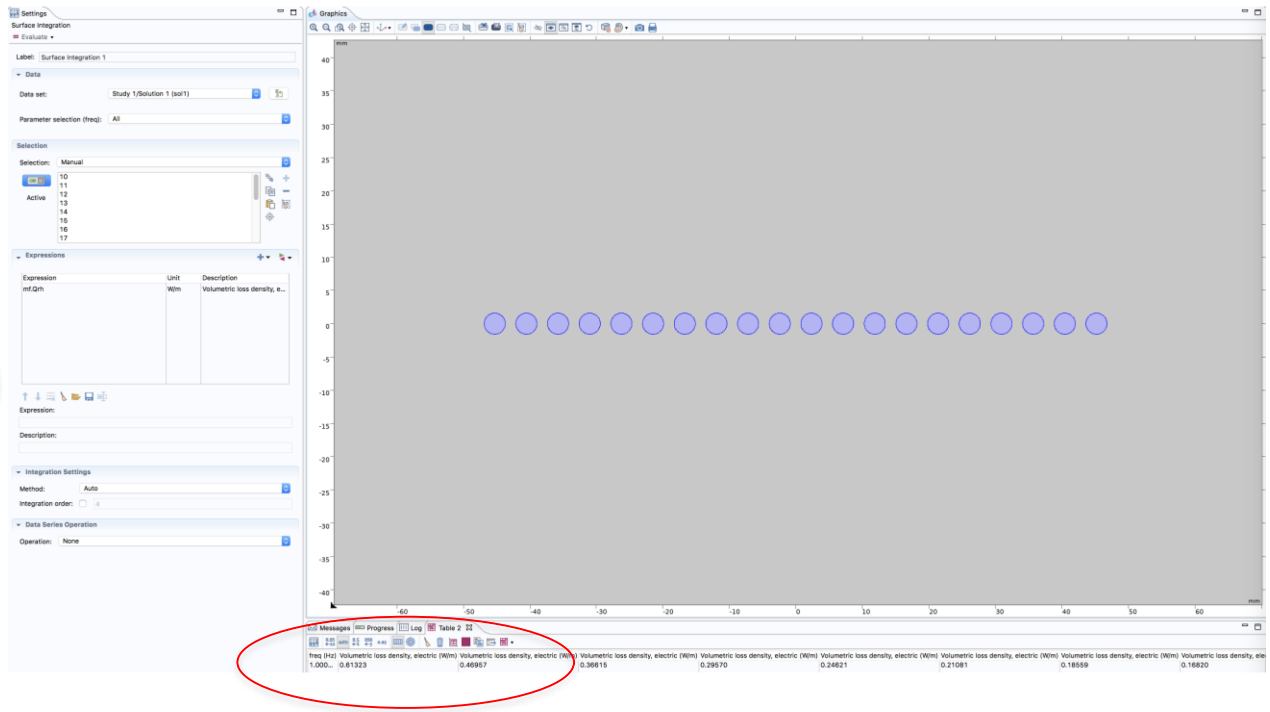
Surface Integration
Over
Selected Area

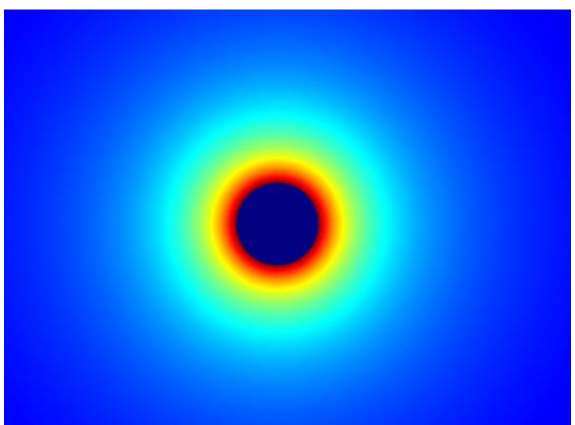
mf.Qrh

Calculation of Normalized Proximity Resistance: Rp/Ro – COMSOL® via: mf.Qrh

"Surface Integration"
Over
Area of Each Wire

$$\frac{R_p}{R_o} = \frac{(mf.Qrh)_{xx\text{-}\ Wire}}{(mf.Qrh)_{1\text{-}\ Wire}}$$





Ref: Single Wire R_o

Example: $$\left.\frac{R_p}{R_o}\right|^{[W3of20]} = \frac{0.22470}{0.1289} = 1.7432$$

Comparison: Analytic vs COMSOL®

3-Wire Proximity Loss Factor, (c/a)= 2.0: R_p/R_o

Analytic: Mathematica®

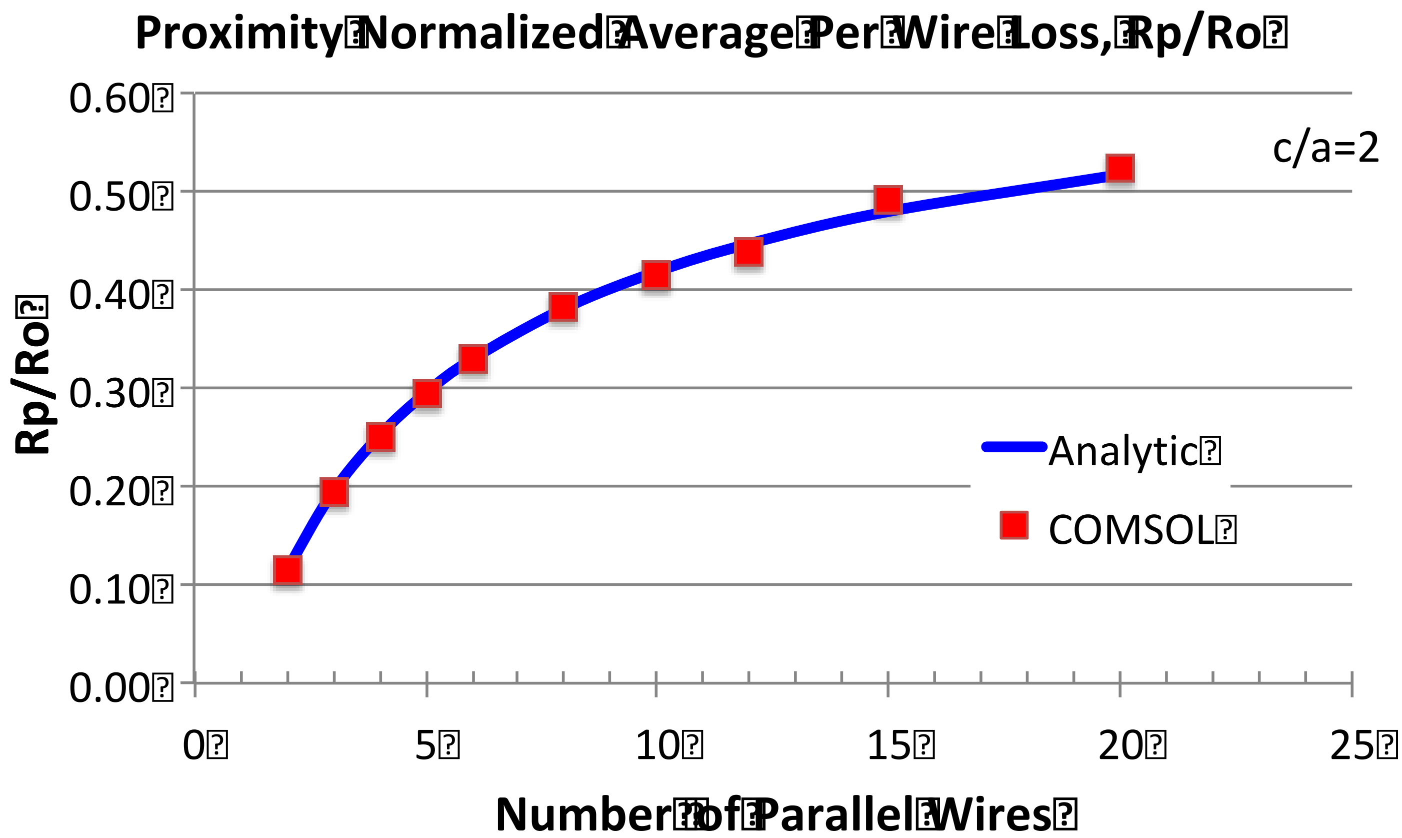
Simulation: COMSOL®

Method	Rp/Ro outer	Rp/Ro center	Rp/Ro (ave of 3-wires)
Theory	0.4986	0.039	0.3455
COMSOL®	0.4968	0.0391	0.3443

Excellent Agreement within 0.5%

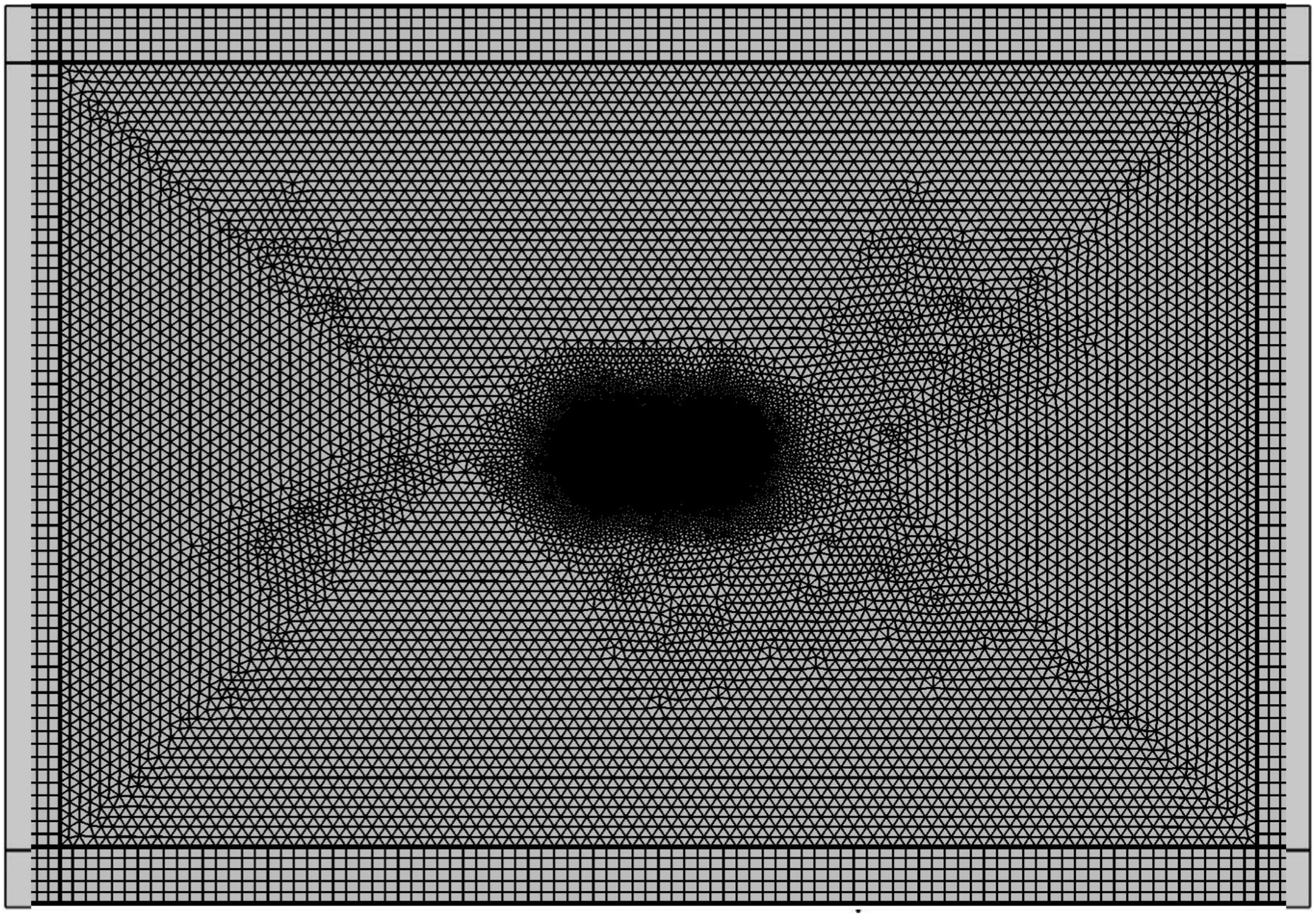
Comparison: Analytic vs COMSOL®

N-Wire Average Proximity Loss Factor: $(R_p/R_o)_{ave}$

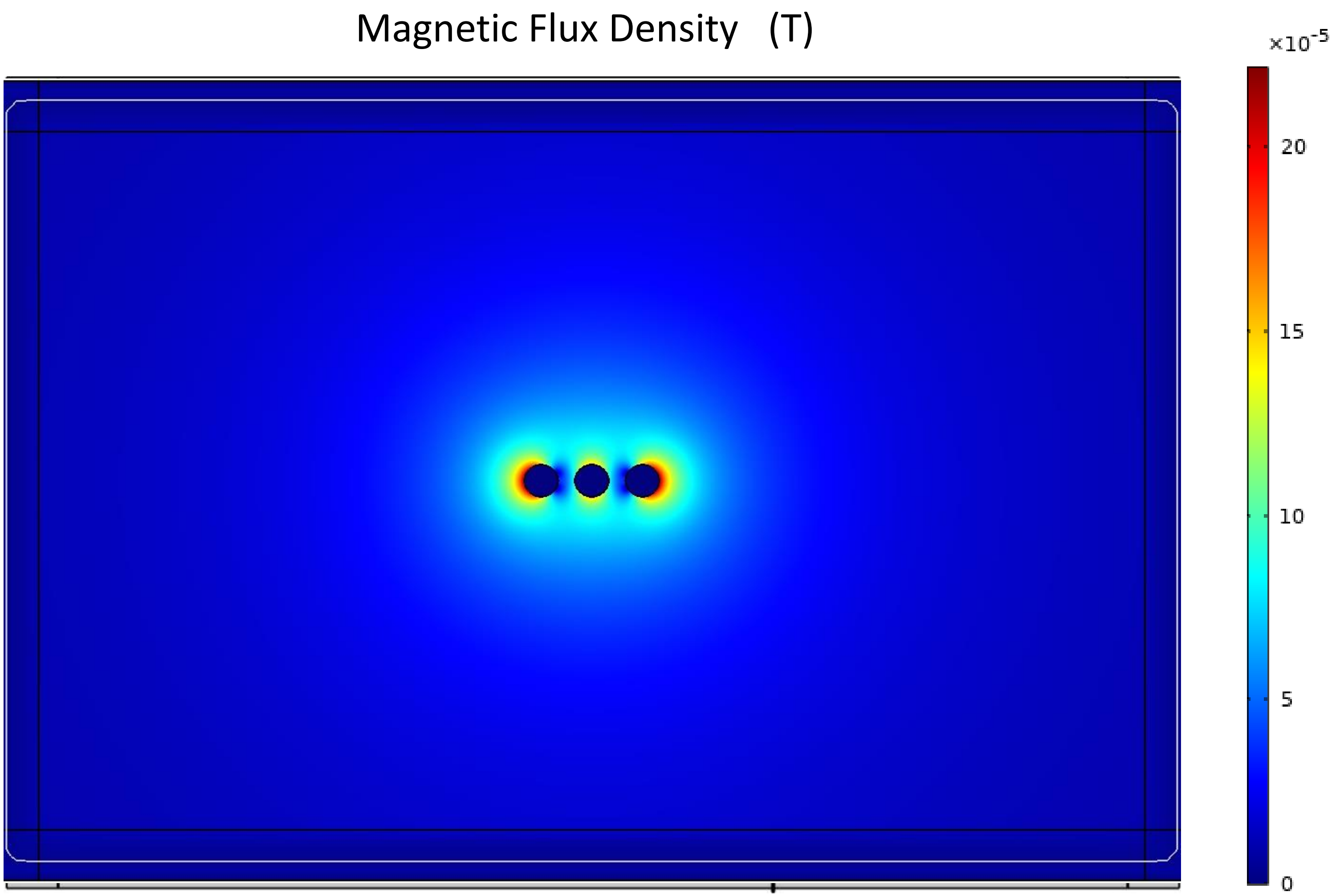


Above 10 wires simulations: greater errors due to mesh area outside wires

3 Wire Mesh Boundary - Example



3 Wire Mesh Boundary - Example



Findings from Studies

- Proximity: A Good "Challenge" Analytic Solution Problem
  - Proximity Effect for Parallel Wires
    - First Solutions 1972, now extended to more wires
  - Excellent Agreement; Theory vs Simulated
    - Great care to accurately represent problem needed

- Mathematica® Analytic Solutions
  - Require adequate terms to converge (troubles for very small spacing)
  - Yields both: current distributions and added losses

- COMSOL® Simulation Solutions
  - Require very careful meshing for accurate solution
    - Large region to external boundary
  - Careful post-processing to obtain losses
    - Loss best done via mf.Qrh

Conclusions – Proximity Problem

- Good Agreement: Analytic and Simulations
  - Requires careful meshing
    - Extra mesh-points in region of rapid field changes
    - External boundary needs to be "far" away
  - Requires careful number of analytic terms
    - Typically 6 to 8 terms is sufficient
- Proximity Effect Results:
  - Severity of added resistance increases with number of wires
  - Severity of added resistance increases for smaller wire spacing
  - Center region wires with many wires less severe change
- Future:
  - Might provide a common "calibration" problem
    - Could use agreed values as reference
  - Possibly a useful means to improve auto meshing
    - Try to improve meshing dynamic range

Example Future Simulation Evaluations via Proximity

- Mesh Effects
  - Quantify accuracy versus mesh density
  - Quantify required mesh density versus field gradient
  - Quantify "infinite" boundary effects
  - Quantify required distance and meshing at "far" boundary region
    - Ex: minimum boundary = 5 x largest object dimension

- Post-Processing:
  - Improve analysis to quantify losses
  - Improve quantification of field gradients

Thank You

Acknowledge: Very valuable partial support for this work by ProlecGE