The Proximity Effect: A Comparison of COMSOL® and Analytic Solutions J. A.

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INTRODUCTION: In this paper we use the proximity effect for parallel wires as a means to evaluate software simulations such as COMSOL® with AC/DC Module solver against a new analytic solution. This 2D problem is very challenging as it includes:

1. Very high dynamic range of fields
2. Very thin regions of spacially varying currents
3. Difficult post-processing for bulk resistance

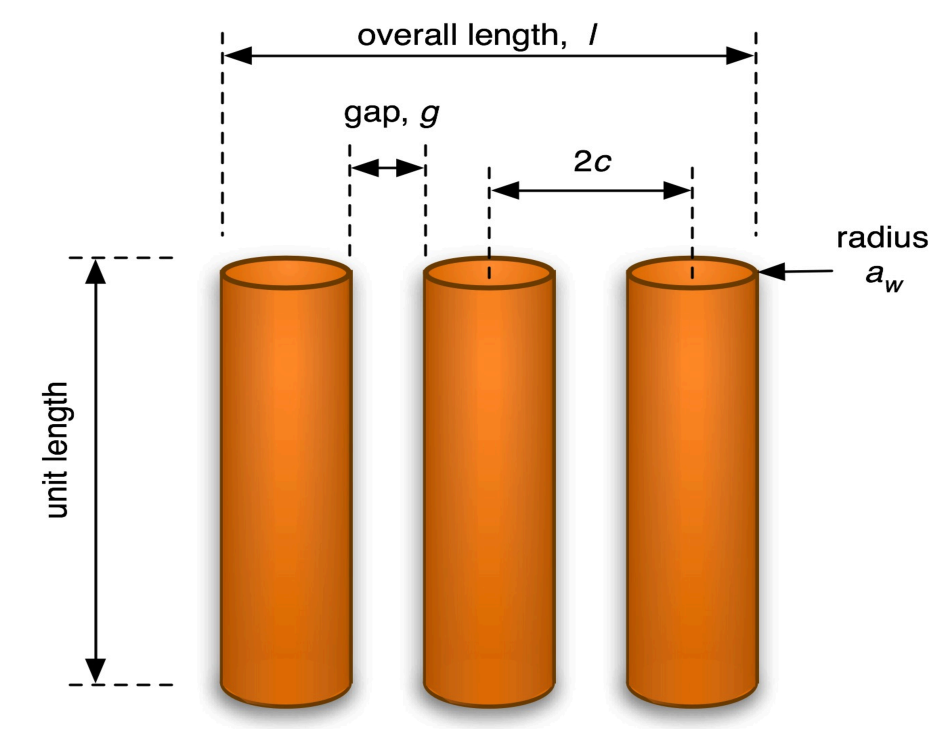

Figure 1. Parallel Wire Geometry

COMPUTATIONAL METHODS: Both analytic Mathematica® and COMSOL® with AC/DC Module solvers were used.

Analytic Solution: Employed magnetic vector potentials for surface currents to obtain a set of integral equations of the form for the surface current distribution, $g_m(\theta)$:

$$g_m(\theta) = 1 + \frac{1}{\pi}\int_{-\pi}^{\pi}\left(\sum_{l=1,l\neq m}^{m} g_l(\theta')K_{ml}\left(\theta,\theta'\right)d\theta'\right).$$

with

$$K_{ml}\left(\theta,\theta'\right) = \frac{1+2c/a(m-l)\cos\theta - \cos(\theta-\theta')}{(r'_{ml})^2},$$

The integration can be performed exactly by use of Cauchy's Integral Theorem. [3] After integrating exactly, the orthogonality property of cosines can be used for simplification. The calculation was implemented in Mathematica®.

COMSOL® Solution: An example grid with very fine spacing for high variation regions is shown in Figure 2.

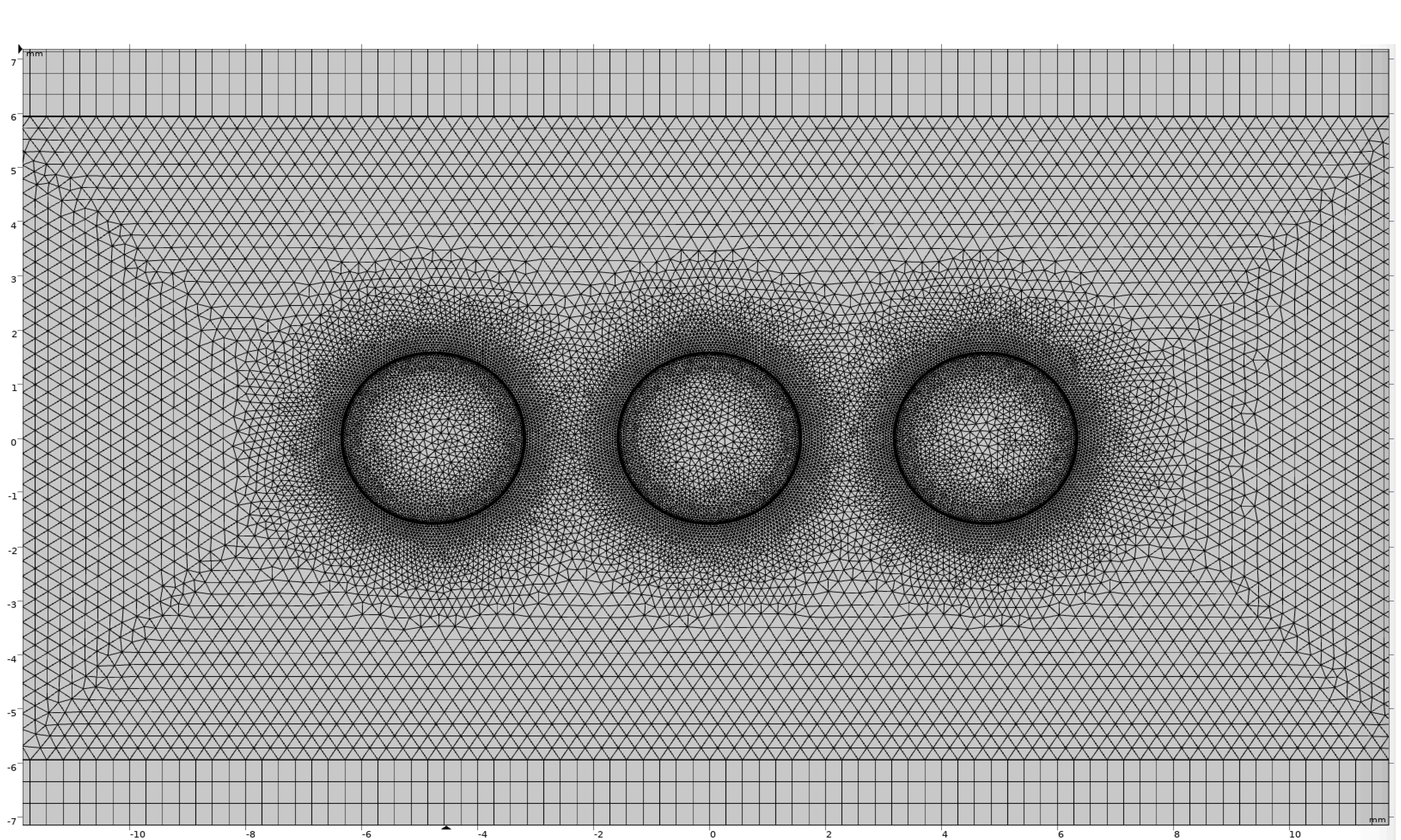

Figure 2 Highly Localized Grid Used for 3 Parallel Wire Solution

Example Results: The proximity effect redistribution of surface charges is shown in the COMSOL® magnetic flux density plot of Figure 3. Table 1 and Figures 4 and 5 compare the two solution methods.

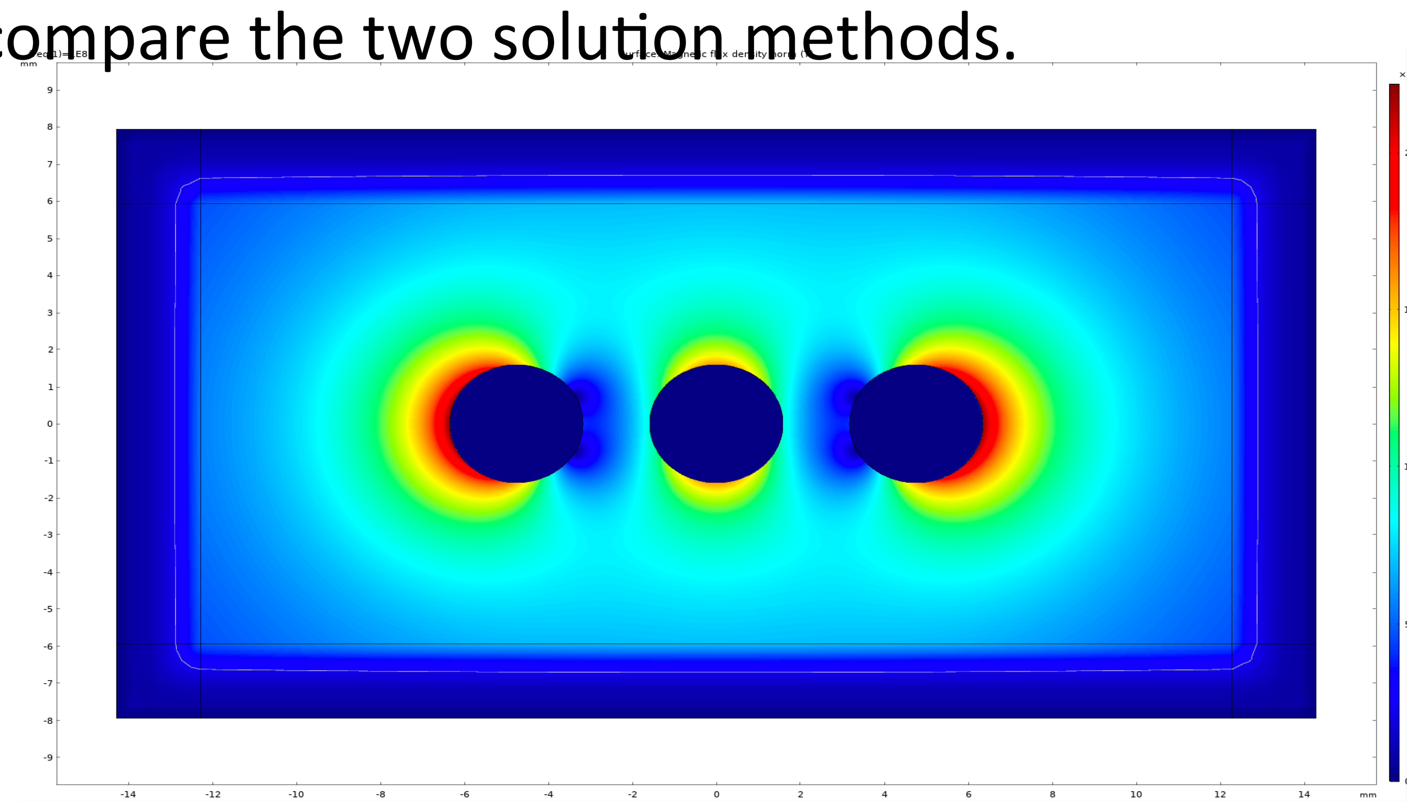

Figure 3. Magnetic Flux Density 3-Wires, 1-ampere, c/a=1.5

Method	Rp/Ro outer	Rp/Ro center	Rp/Ro Ave (3-wiress)
Theory	0.4986	0.0390	0.3455
COMSOL®	0.4968	0.0391	0.3443

Table 1 Comparison Theory-Mathematica vs COMSOL-AC/DC 3-Parallel Wires with c/a = 2.00

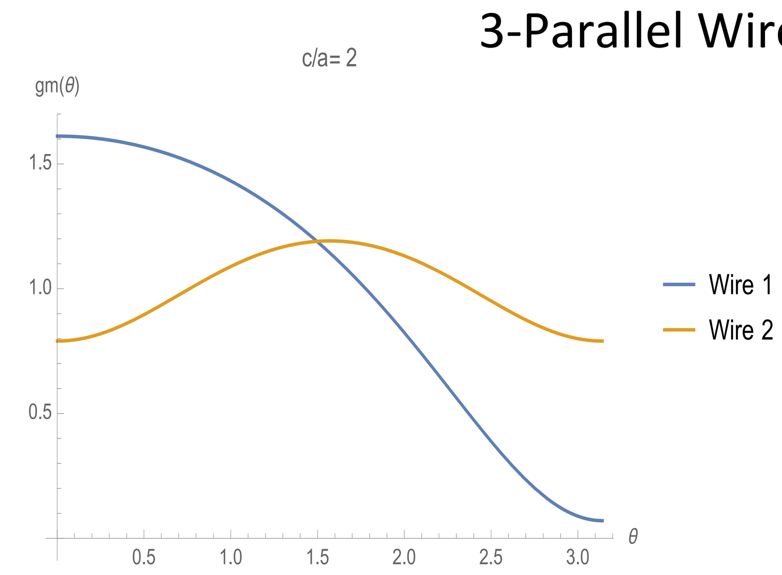

Figure 4. 3-Wire Analytic Mathematica® Solution c/a = 2.00

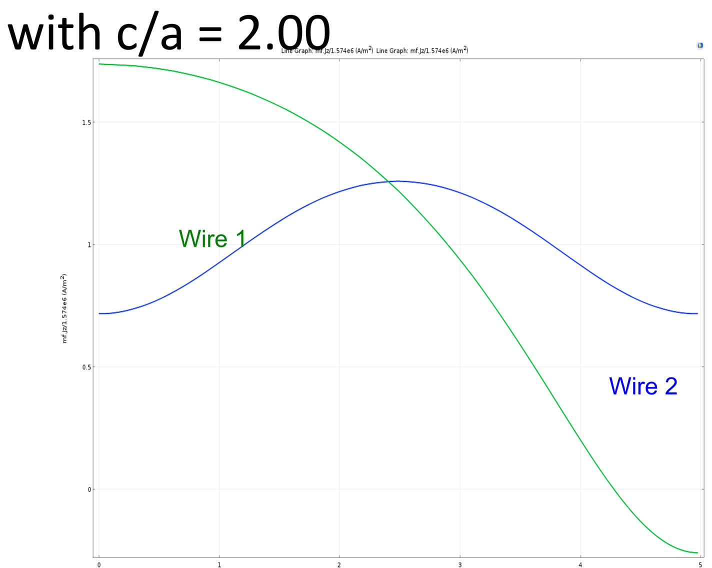

Figure 5. 3-Wire Simulation COMSOL® Solution c/a = 2.00

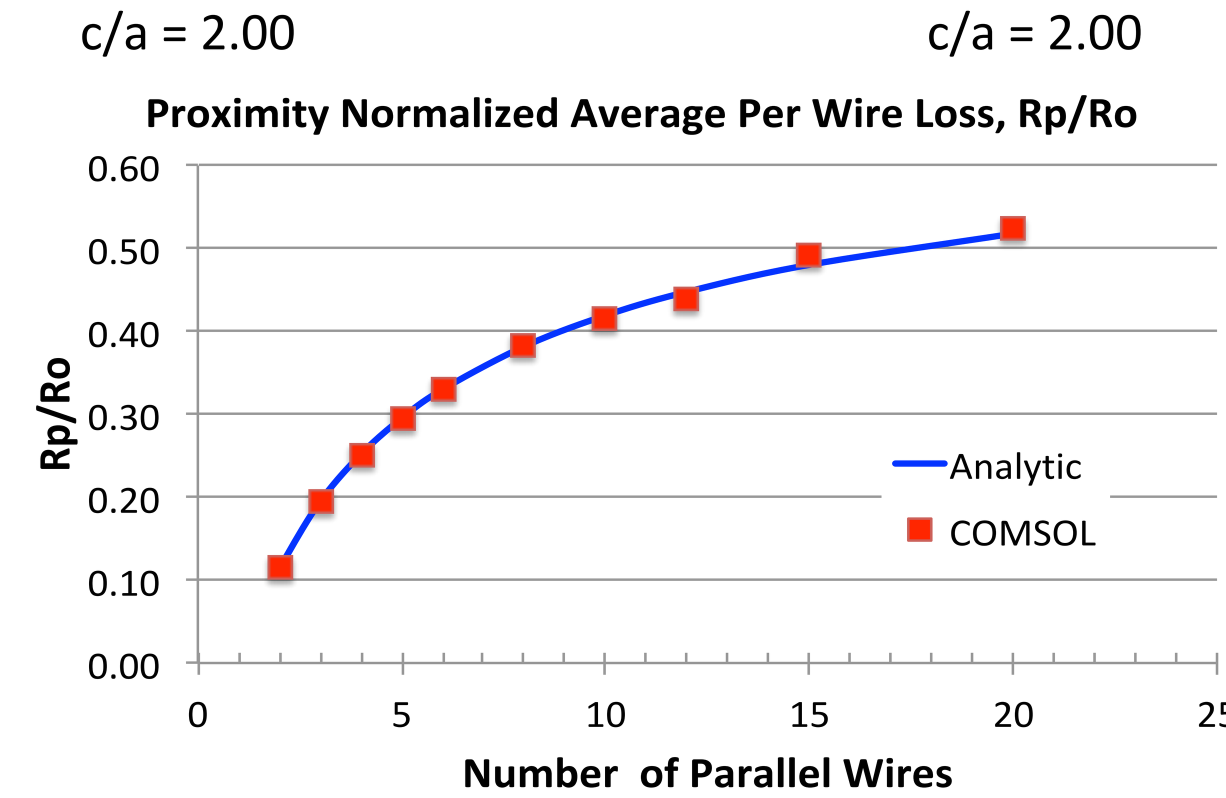

Figure 6. Comparison Theory-Mathematica vs COMSOL-AC/DC N-Parallel Wires with c/a = 2.00

CONCLUSIONS: The proximity effect analytic solution provides a good opportunity to "validate" EM simulation software such as COMSOL®. It challenges both field solutions as well as post-processing to quantify net resistances in the face of extreme variations.

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