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# Vortex Formation and Oscillations in Flows Through a Sudden Expansion in a Pipe

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**INTRODUCTION**: A classic problem in process engineering is the estimation of the excess frictional losses that occur as a fluid passes through a sudden expansion. At elevated Reynolds numbers (Re=Ud/$\nu$) there are simple correlations that provide reasonable estimates of the pressure drop across an expansion.

However in the range 10<Re<1000, this situation is more complex due to the appearance of several inertial instabilities that occur as the fluid approaches the critical Re for turbulent flow. This includes the initial appearance of a radially symmetric vortex adjacent to the entrance of the expansion at very low Re, followed by an unstable transition to an asymmetric vortex and, finally, an unstable transition to an oscillatory flow at higher, laminar Re.

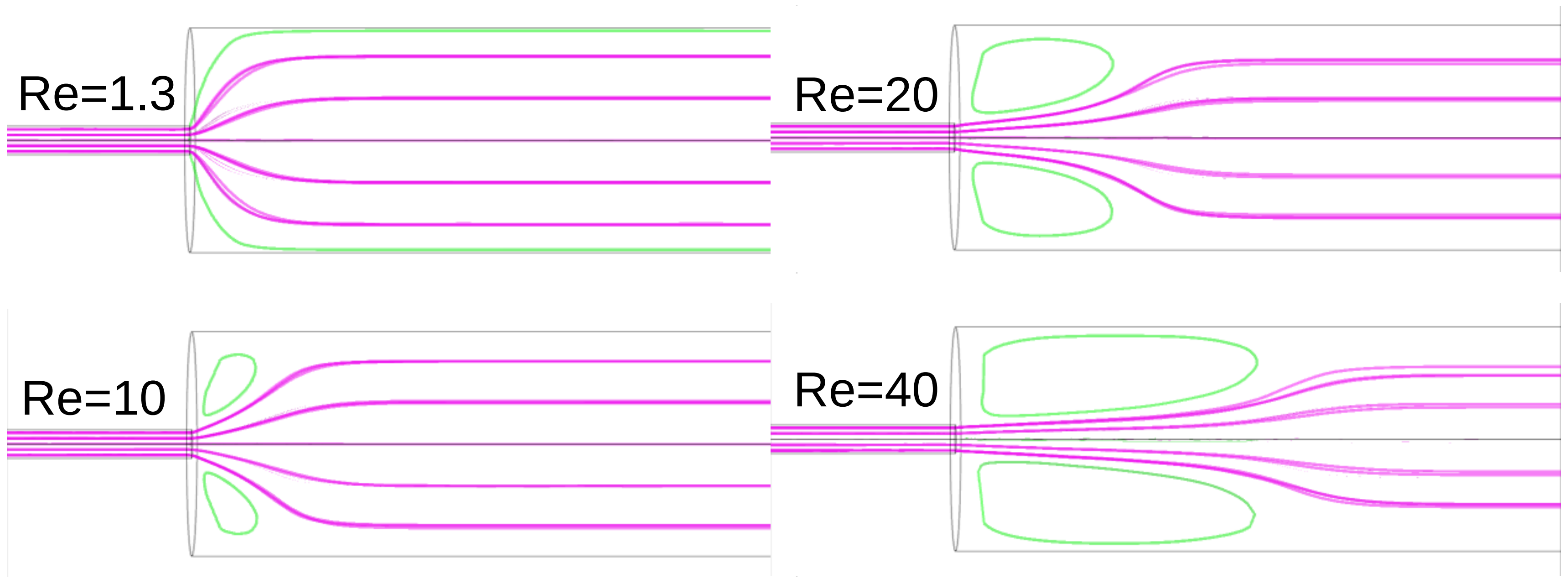

**Figure 1**. ***Top Left***: At low Re, the flow profile is symmetric and has no vortices. ***Bottom Left:*** Symmetric vortex appears at Re~6… ***Top Right:*** …and increases linearly with increasing Re… ***Bottom Right:*** …while maintaining its symmetry.

**COMPUTATIONAL METHODS**: All of the flows shown on this poster were generated using [1] the Navier-Stokes equations together with [2] the Continuity equation$^2$ as provided in the single phase laminar flow physics of COMSOL's CFD module.

$$\rho\frac{\partial \boldsymbol{u}}{\partial t} + \rho\boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla p + \nabla\cdot\mu\nabla\boldsymbol{u} + \rho\boldsymbol{g} \qquad [1]$$

$$\frac{\partial \rho}{\partial t} + \nabla\cdot\rho\boldsymbol{u} = \boldsymbol{0} \qquad [2]$$

The equations were solved together with the "no-slip" condition on all solid surfaces with a fixed inlet flow rate and zero gauge pressure at the outlet. In all cases the initial conditions specified zero velocity and gauge pressure throughout the computational domain.

In order to visualize the first and second hydrodynamic instabilities, a time-dependent study was used with small time increments to allow the instability to emerge from the base flow.

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**RESULTS**: Fig. 2 shows the emergence of the first hydrodynamic instability at Re~108 which manifests as a steady asymmetric flow in the tube. The next instability arises near Re~140 and manifests as an oscillatory flow with a frequency that increases with

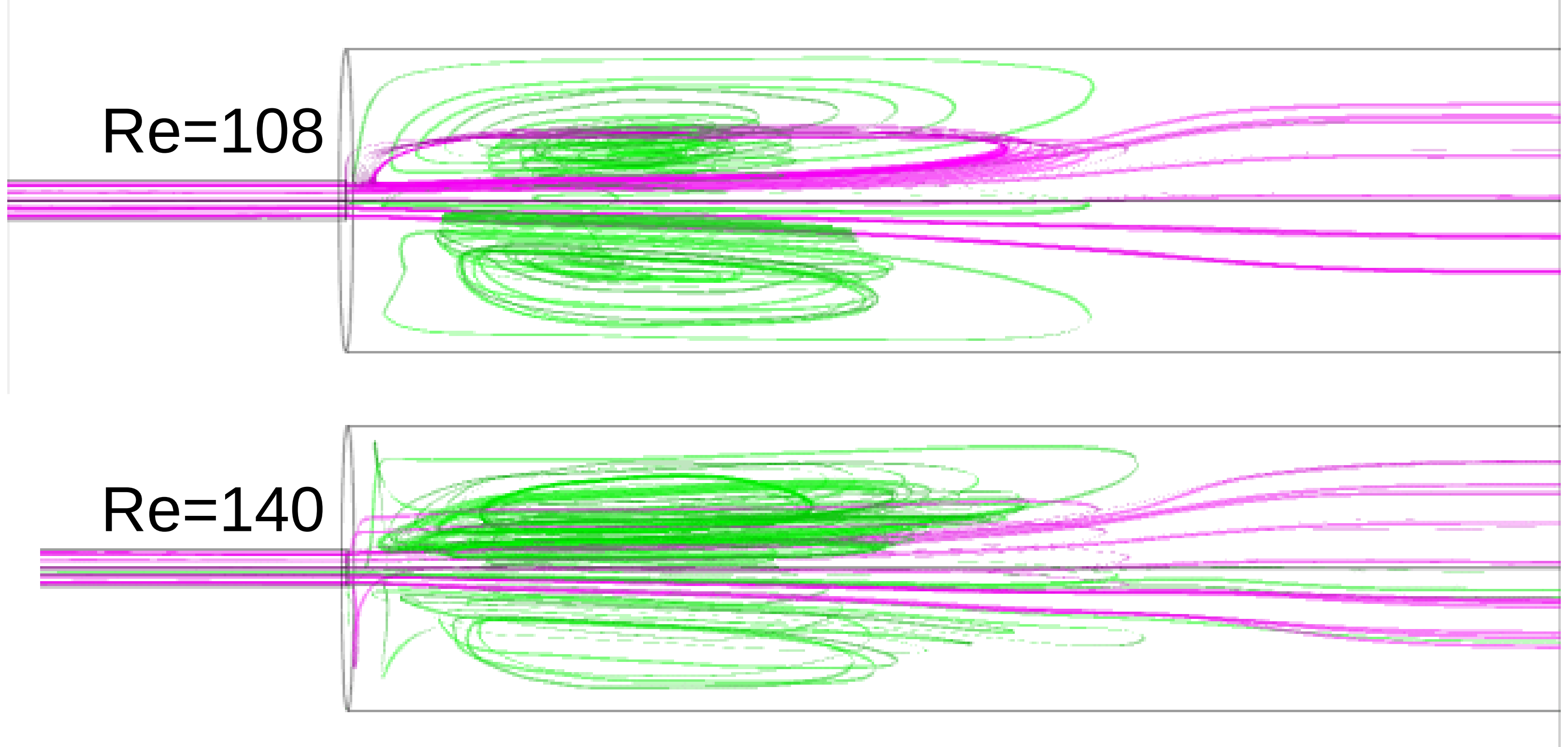

**Figure 2**. ***Top:*** The first hydrodynamic instability, which is an exchange of stationary states from a symmetric to a non-symmetric profile can be seen at Re near 108. ***Bottom:*** Near Re=140, a small oscillation sets in which, unfortunately, cannot be gleaned from a static graphic.

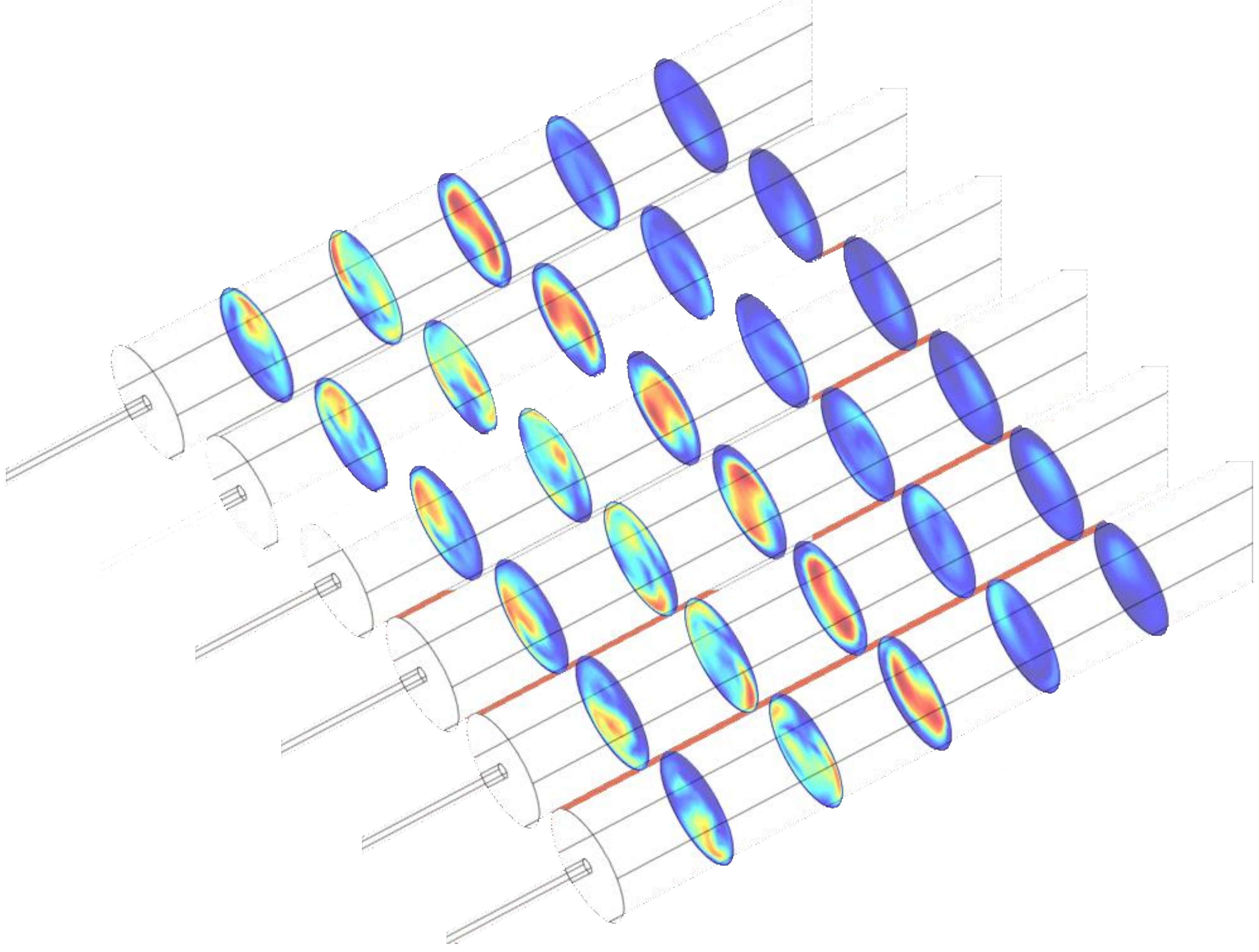

**Figure 3**. At Re~450, timed snapshots of the magnitude of the velocity in the cut planes taken 0.05 seconds apart show the oscillation as a counter-clockwise rotation of the flow field. It takes about 0.5 seconds to complete a 360° rotation of the field.

**CONCLUSION**: Hydrodynamic instabilities that can be modeled using the equations of conservation of momentum can be readily simulated using the CFD module in COMSOL Multiphysics® v5 so long as the bifurcating flows are laminar. Generally, these simulations take less than an hour to set up; and 3D problems run to completion in several hours, depending on their DOF.

**REFERENCES**:

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