Numerical simulation of a 4 K hybrid refrigerator combining GM gas expansion effect with magnetic refrigeration effect

X.H. Guo[1,2], K. Li[1], W. Dai[1,2], X.Q. Gao[1], J. Shen[1,2], W.C. Gong[1,2]

1. Key Laboratory of Cryogenics, Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, Beijing 100190, China
2. University of Chinese Academy of Sciences, Beijing 100049, China

1. Introduction

Gifford-McMahon (GM) refrigerator based on gas expansion effect can provide a cooling temperature below 4.2 K. Because of the limited specific heat capacity of regenerator materials as well as non-ideal properties of helium at this temperature, the efficiency of GM refrigerator can only reach about 1% of the ideal thermodynamic efficiency. Considering the intrinsic high efficiency of magnetic refrigeration, an integration of both effects may lead to a higher efficiency. And some recent experiments have confirmed this. But it is difficult to analyze the physical mechanism only by observing these experimental results. So we developed a 2D axis-symmetric and transient numerical model to study and optimal the hybrid refrigeration. In this paper, the timing between gas expansion and magnetic refrigeration is studied, and the corresponding cooling power and efficiency are obtained and compared with the results from pure gas expansion. Meanwhile, the modified thermal conductivity of porous media in COMSOL is proposed for a more accurate estimation of cooling power.

2. Simulation model

Model description:

The hybrid refrigeration cycle consists of four processes: helium expansion, cold-to-hot blow/magnetization, helium compression, hot-to-cold blow/demagnetization. The model includes a porous second stage regenerator filled with Pb and ErNi and an expansion chamber. A periodic sinusoidal magnetic field is applied to the regenerator section that contains ErNi. Moving mesh is used to describe periodic sinusoidal displacement at the bottom of the expansion space.

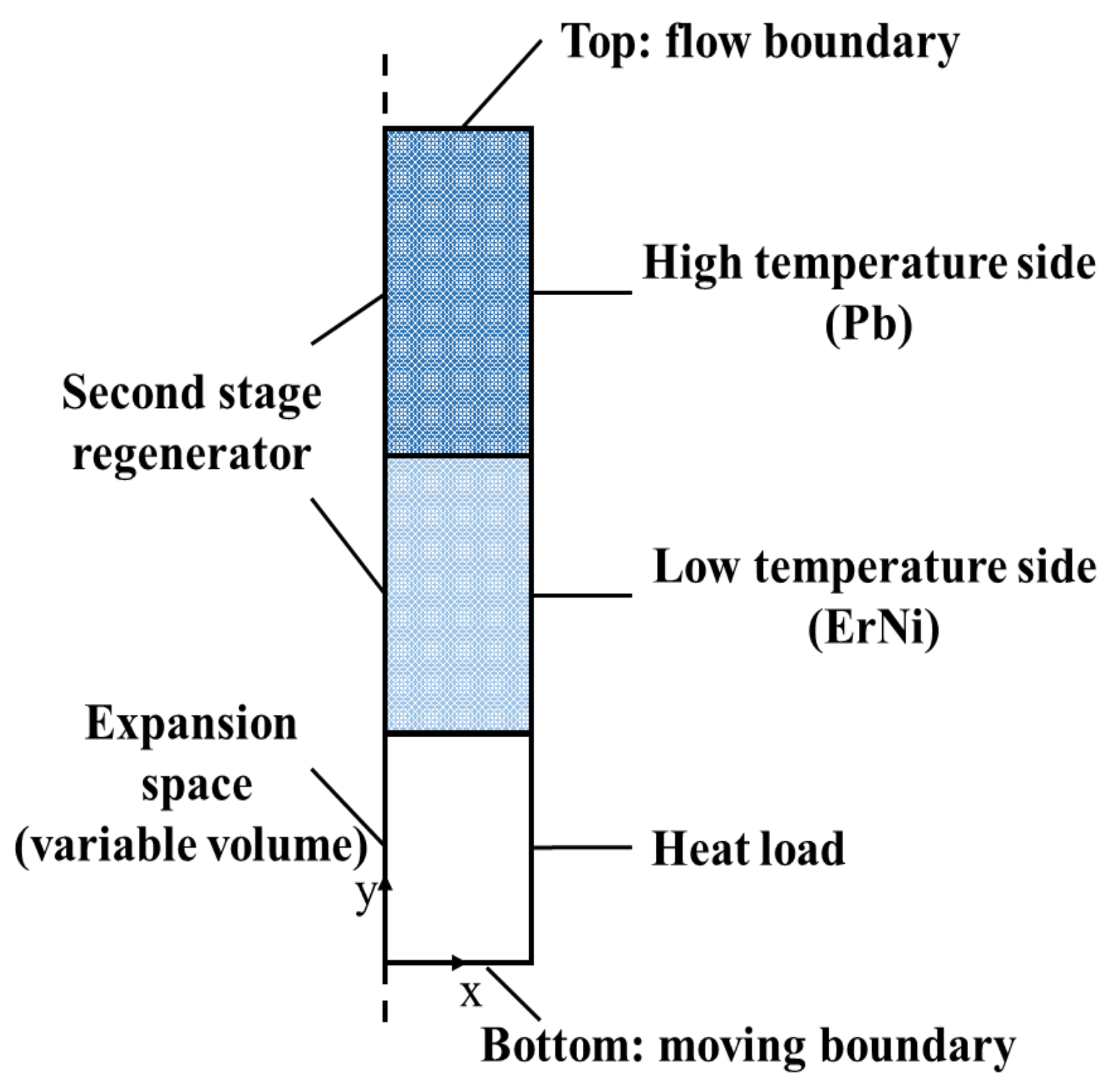

Fig. 1 Geometric model

parameters	value
Regenerator diameter	30 mm
Particle diameter	0.4 mm
Mass of Pb	325 g
Mass of ErNi	235 g
Stroke	35 mm
Inlet temperature (T_0)	45 K
Inlet pressure (p_0(t))	0.6-2.4 MPa
Frequency	1 Hz
Magnetic field	0-1.1 T(sine)

Table 1 Geometric and operating parameters

Governing equations：

Continuity equation, momentum equation, energy equations in expansion chamber and regenerator are as following:

$$\frac{\partial \rho_{\mathrm{f}}(T_{\mathrm{f}},p)}{\partial t}+\nabla\cdot(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)\mathbf{u})=0$$

$$\frac{\partial(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)\mathbf{u})}{\partial t}+(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)\mathbf{u}\cdot\nabla)\mathbf{u}+\nabla p-\nabla^2(\mu_{\mathrm{f}}(T_{\mathrm{f}},p)\mathbf{u})=0$$

$$\frac{\partial\big(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)c_{p,\mathrm{f}}(T_{\mathrm{f}},p)T_{\mathrm{f}}\big)}{\partial t}+\mathbf{u}\cdot\nabla\big(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)c_{p,\mathrm{f}}(T_{\mathrm{f}},p)T_{\mathrm{f}}\big)=\nabla\cdot(k_{\mathrm{f}}(T_{\mathrm{f}},p)\nabla T_{\mathrm{f}})$$

$$\frac{\partial\big(\varepsilon\rho_{\mathrm{f}}(T_{\mathrm{f}},p)c_{p,\mathrm{f}}(T_{\mathrm{f}},p)T_{\mathrm{f}}\big)}{\partial t}+\varepsilon\mathbf{u}\cdot\nabla\big(\rho_{\mathrm{f}}(T_{\mathrm{f}},p)c_{p,\mathrm{f}}(T_{\mathrm{f}},p)T_{\mathrm{f}}\big)=\nabla\cdot(\varepsilon k_{\mathrm{f}}(T_{\mathrm{f}},p)\nabla T_{\mathrm{f}})+q_{\mathrm{sf}}(T_{\mathrm{s}}-T_{\mathrm{f}})$$

$$\frac{\partial\Big((1-\varepsilon)\rho_{\mathrm{s}}c_{p,\mathrm{s}}(T_{\mathrm{s}},\mu_0 H)T_{\mathrm{s}}\Big)}{\partial t}=\nabla\cdot\Big((1-\varepsilon)k_{\mathrm{s,eff}}(T_{\mathrm{s}})\nabla T_{\mathrm{s}}\Big)+q_{\mathrm{sf}}(T_{\mathrm{f}}-T_{\mathrm{s}})+\dot{Q}_{\mathrm{MCE}}$$

Where, the modified thermal conductivity *k*s,eff(*T*s) of porous media is proposed, which is important for a more accurate estimation of the cooling power for such kind of small-scale liquid helium temperature refrigerator.

Boundary conditions：

The top of the regenerator is a pressure inlet, the bottom of the expansion chamber is a periodic reciprocating motion boundary using moving mesh to describe the volume change of expansion chamber. The others are no-slid and adiabatic wall. The inlet pressure and the displacement at the bottom of expansion space curves as shown in Fig. 2.

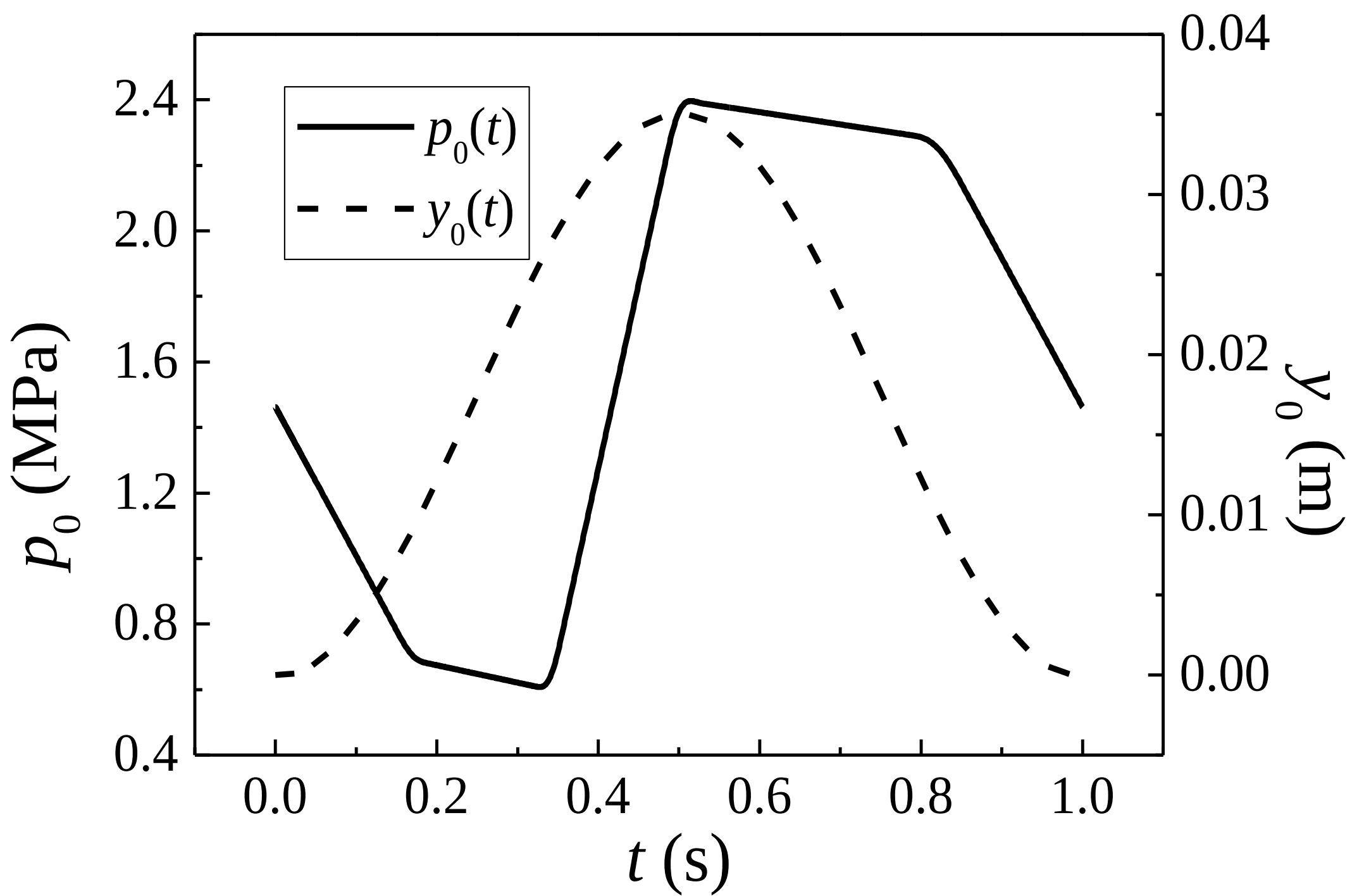

Fig. 2 Boundary conditions of pressure (*p*0(t)) and displacement (y0(t))

3. The results

To describe the timing between gas expansion and magnetic refrigeration, a phase angle is defined as the phase difference between the maximum value of varying magnetic field and the minimum value of varying expansion chamber volume.

Fig. 3 shows numerical and experimental results of the no-load temperature at different phase angles. The numerical results show that the optimal phase angle is 90° , and the corresponding no-load temperature reaches 2.45 K.

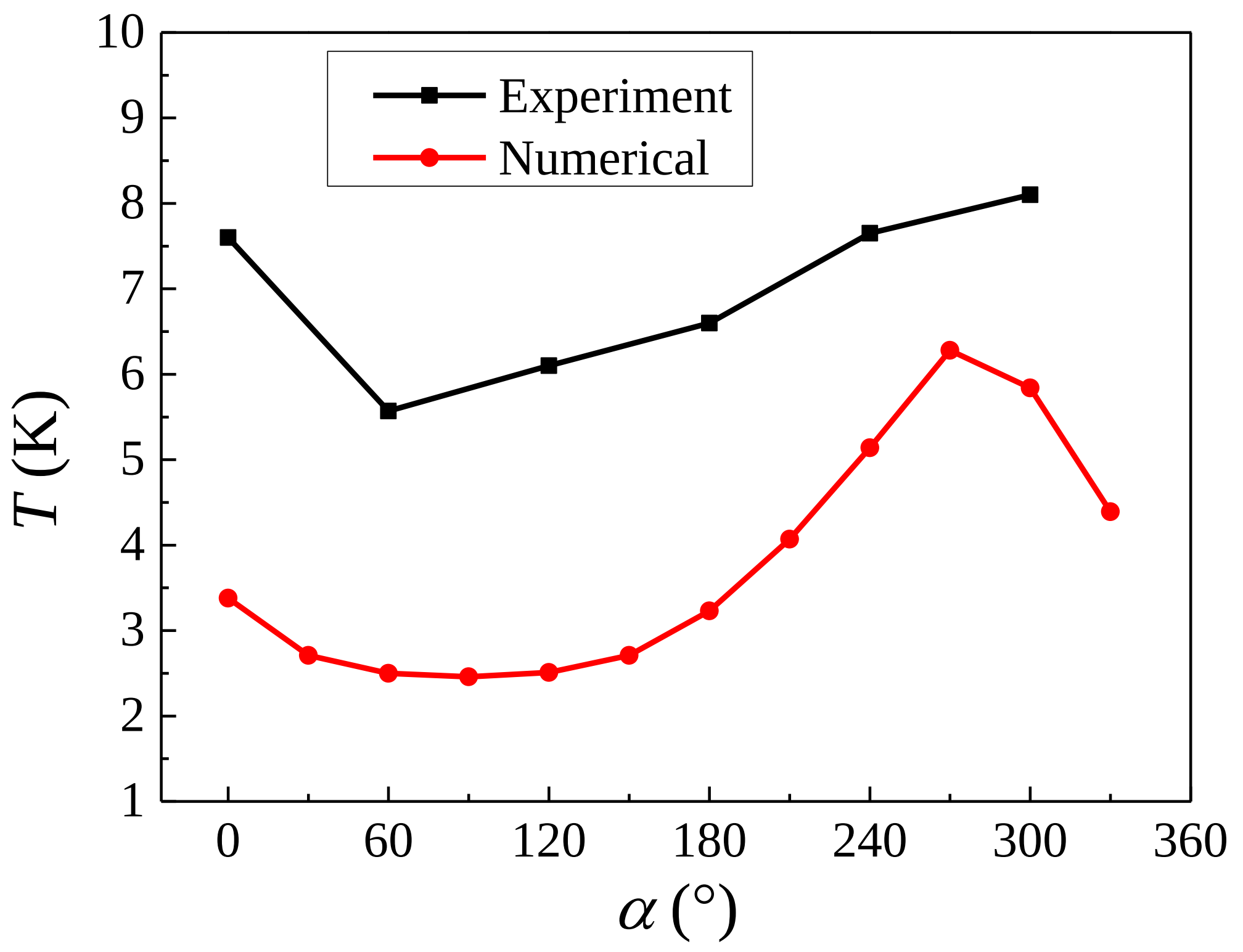

Fig. 3 No-load temperature at different phase angles

Fig. 4 and Fig. 5 respectively shows the cooling power and the efficiency of the hybrid refrigerator at optimal phase angle as well as the GM as a function of temperature. As the temperature increases, the cooling power and the efficiency increase. The cooling power at 4 K increased from 0.13 W with pure gas expansion to 0.27 W with the hybrid scheme, and the efficiency improved by 2.13 times.

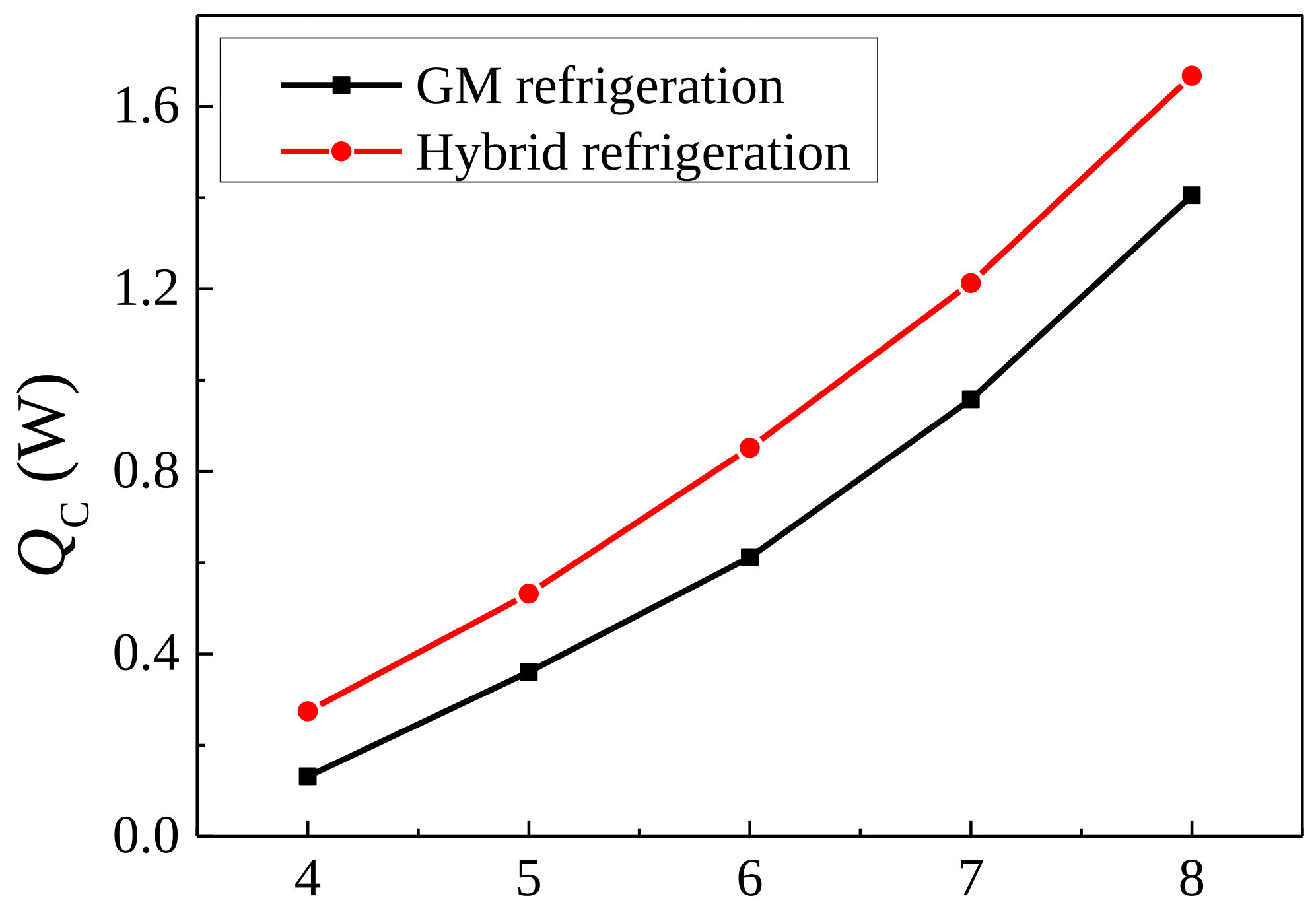

Fig. 4 Cooling power at different temperatures

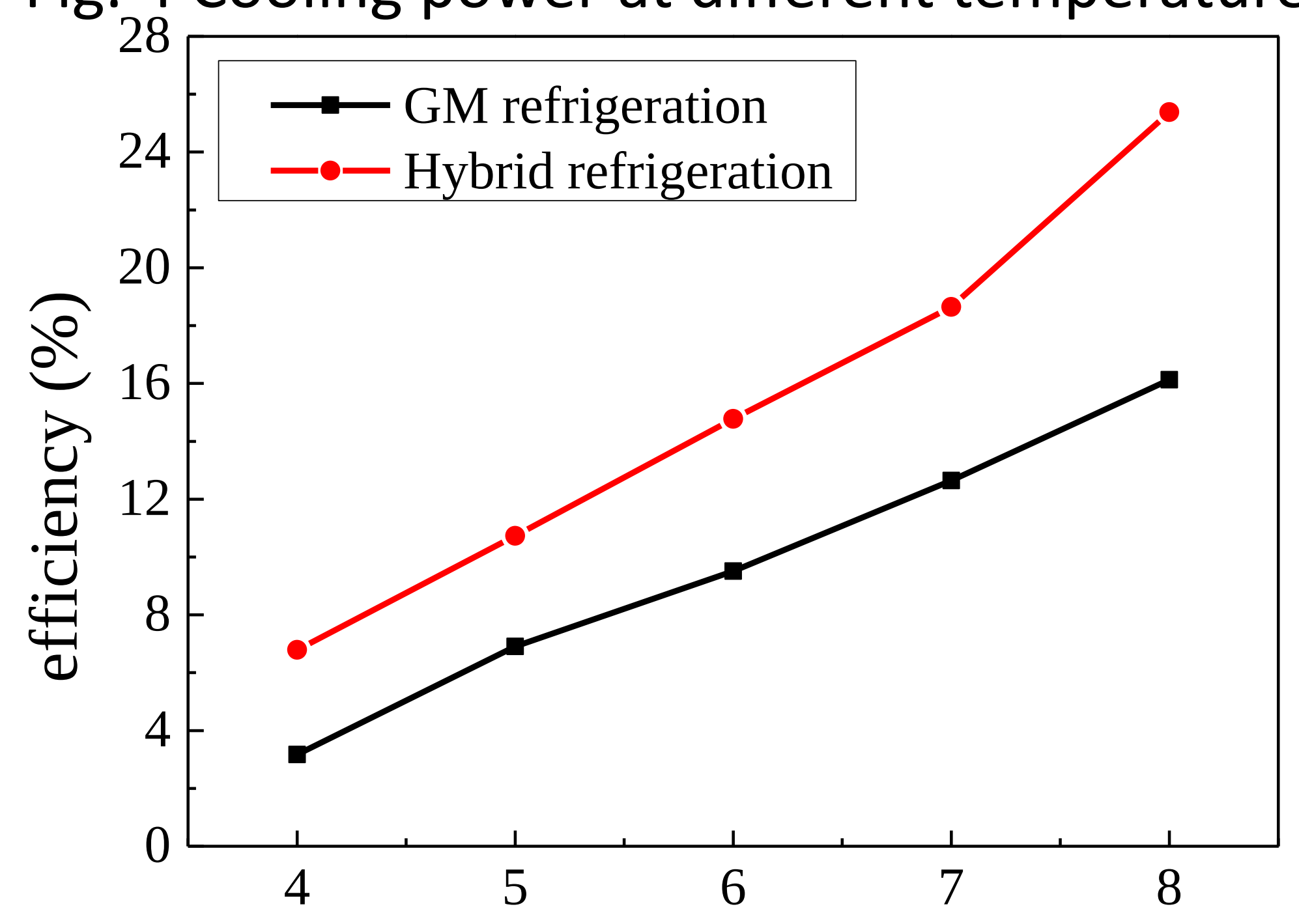

Fig. 5 Efficiency at different temperatures

4. Conclusion

- **The optimal phase angle of hybrid refrigerator is 90° .**
- **The cooling power and efficiency of hybrid refrigeration at 4 K are respectively 2.07 times and 2.13 times of GM.**
- **This numerical simulation provides guidance for optimizing the performance of hybrid refrigerator.**

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