Empirical Verification of COMSOL-Simulation of Resonance Frequency of an Archimedean Spiral Coil

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Abstract: An simple analytical model for the calculation of the resonance frequency of coils typical consists of a network of inductance, capacitance and resistance. Calculating these three parameters based on spatial geometry is not easy in most cases. The finite element method as used in COMSOL Multiphysics therefore lends itself to approximating a solution to complex structures. In this paper the Radio Frequency module in its 3D Electromagnetic Waves, Frequency Domain formulation is used. The easiest way to calculate the resonance frequency is to use the Eigenfrequency study. In this case it should be noted that only external effects are considered. Influences based on the current distribution in the wire are neglected. In this work, the resonance frequency of an self-made Archimedean spiral coil was measured with a network analyzer to question and verify the simulation results of the COMSOL RF module. The simulation model of the coil was gradually adjusted closer to the geometry of the real coil to distinguish between partial influences. In addition, a parameter sweep was used to study the variation of wire radius, wire spacing, and relative permittivity.

Keywords: Archimedean Spiral Coil, Resonance Frequency

1. Introduction

For analytical calculation of the resonance frequency of coils, parameters such as inductance, capacitance and resistance must be known. These parameters have to be calculated from the geometry of the coil, which generally is not straightforward. Especially calculating the capacitance is very problematic. Due to facts like these, numerical mathematics is used in modern engineering to investigate complex problems and to optimize technical structures by computer simulations. In this work, the resonance frequency of an self-made Archimedean spiral coil was measured with a network analyzer to question and verify the simulation results of the COMSOL RF module.

2. Experimental Coil and Empirical Measurements

2.1 Manufacturing of the Experimental Coil

To create an Archimedean spiral coil, a template was first created using the computer algebra software *Geogebra*. For this purpose, the parameter representation (1) of the Archimedean spiral was used, where d is the average wire distance, N the number of turns, a the starting parameter, b the end parameter, t_0 azimuthal shift and R_A the inner radius. Figure 1 shows the template of an Archimedean spiral with specific parameters.

$$\vec{k}(t) = d \cdot t \cdot \begin{pmatrix} \cos\left(2\pi\left(t - t_0\right)\right) \\ \sin\left(2\pi\left(t - t_0\right)\right) \end{pmatrix}, a \leq t \leq b \quad (1)$$

$$t_0 = a - 1 \quad (2)$$

$$a = \frac{R_A}{d} \qquad b = a + N \quad (3)$$

$$\zeta = 2\pi a \qquad \xi = 2\pi b \quad (4)$$

With the parameters (4) and equation (5) the arc length L of the Archimedean spiral is calculated for estimation of the wire length.

$$L = \frac{d}{4\pi}\left(\ln\left(\frac{\sqrt{\xi^2 + 1} + \xi}{\sqrt{\zeta^2 + 1} + \zeta} \right) + \xi\sqrt{\xi^2 + 1} - \zeta\sqrt{\zeta^2 + 1} \right) \quad (5)$$
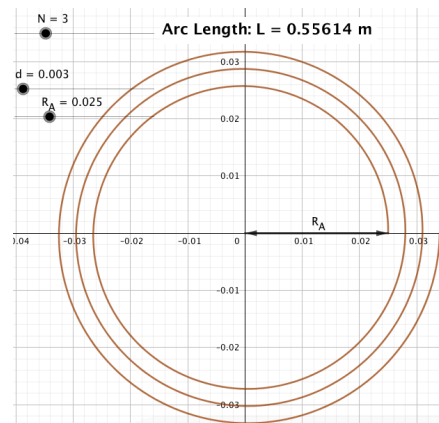

Figure 1. template of the Archimedean spiral coil with N=3 number of turns, inner radius R_A=2.5 cm, average wire distance d = 3 mm and arc length L = 55.614 cm

After sticking the template on cardboard, the copper wire was bent and fixed with needle and thread. Like shown in figure 2 (b) a SMA socket was soldered to the wires as a connector. Figure 2 shows the results of manufacturing in front and side view.
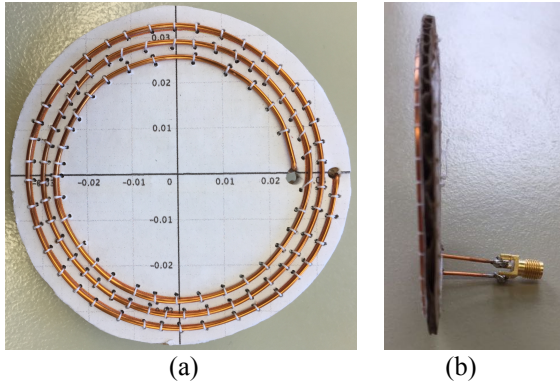
(a)   (b)

Figure 2. Self-made Archimedean spiral coil with N=3 number of turns, inner radius R_A=2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm and wire radius of R = 0.5 mm

2.2 Results of Measurement

A network analyzer from *Rhode & Schwarz* was used to investigate the frequency dependence of the input impedance of the Archimedean spiral coil. Figure 3 shows the result of the measurement. The resonance frequency is about f_R = 100 MHz and the input impedance is 12 kΩ. In the frequency range above the resonance frequency, the coil acts in general as a short-circuit due to capacitive influences.
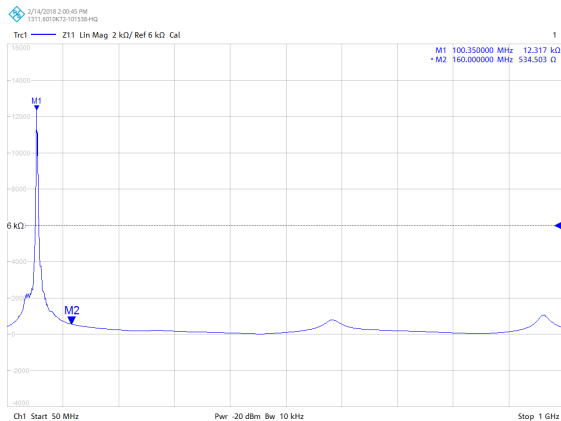

Figure. 3. results of measurement of the input impedance of archimedean spiral coil with N=3 number of turns, inner radius R_A=2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm and wire radius of R = 0.5 mm

3. Use of COMSOL Multiphysics

For the COMSOL simulations the *radio frequency - electromagnetic waves, frequency domain* module with the *eigenfrequency* study was used. In this case COMSOL uses the vectorial maxwell wave equation in its formulation for the electric field, like shown in equations (6) and (7), where ω is the

angular Frequency, σ the material conductivity, ε_0 the permittivity of free space, ε_r the relative permittivity, μ_r the relative permeability, k_0 the free space wavenumber, E the complex electric field vector, λ the complex eigenvalue and δ the attenuation.

$$\vec{0} = \nabla \times \mu_r^{-1}(\nabla \times \underline{\vec{E}}) - k_0^2\left(\varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0}\right)\underline{\vec{E}} \quad (6)$$

$$\lambda = \delta - j\omega \quad (7)$$

If only the spiral is simulated, because of its symmetry it is possible to use a sphere to limit the model. If the model is more detailed, its better to use a cylindrical surface to limit the model volume. The model of the coil had to be built out of a 3D parametric curve and the sweep function for creation of a circular surface around the curve. The materials used in the simulations were air and paper. The problem here was that the permittivity and permeability of the cardboard are unknown. In the properties of the physics the wire had to be removed, cause the eigenvalues are calculated from the outer geometry. Further the boundary condition *perfect magnetic conductor* was used, thus the normal component of the magnetic field at the outer surface is zero.

4. Results of COMSOL-Simulation

4.1 Simulation of an Archimedean Spiral Coil (1)

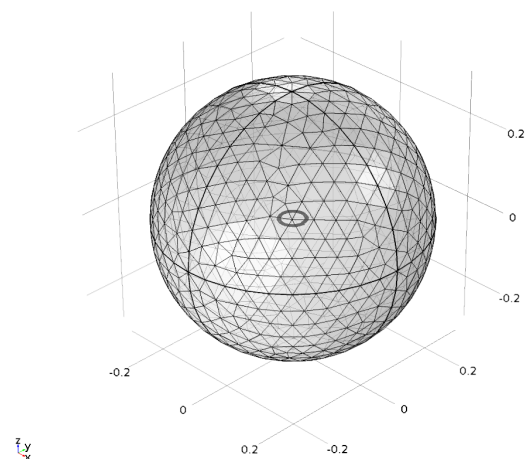In the first simulation only the archimedean spiral coil with a sphere was used as model, like shown in figure 4.

Figure 4. simplified COMSOL-model of the archimedean spiral coil with N = 3 number of turns, inner radius R_A=2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm, wire radius of R = 0.5 mm and sphere radius of 0.3 m

The result of the simulation is shown in figure 5, which is ≈ 55 MHz lower than the measured resonance frequency. Therefore, the model used is still too simplistic to consider all effects. If the radius of the sphere is chosen smaller than 0.3m, the resonance frequency will rise up.
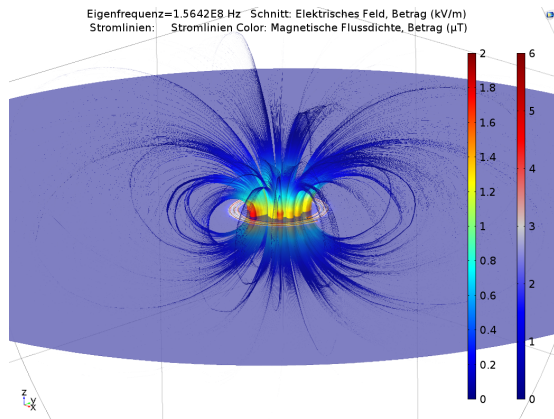

Figure 5. COMSOL-Solution of simplified model of an Archimedian spiral coil. The resonance frequency is $f_R \approx 156$ MHz. The left colorbar displays the norm of the magnetic flux density in µT and the right colorbar the electric field intensity in kV/m.

4.2 Simulation of an Archimedean Spiral Coil (2)

Hence the resonance frequency of the first simulation was far away from the measurement result, a more detailed model was used, which additionally included the connection wires like shown in figure 6. Because of symmetry reduction, in this simulation it's better to use a cylindrical surface for limitation of the model volume.
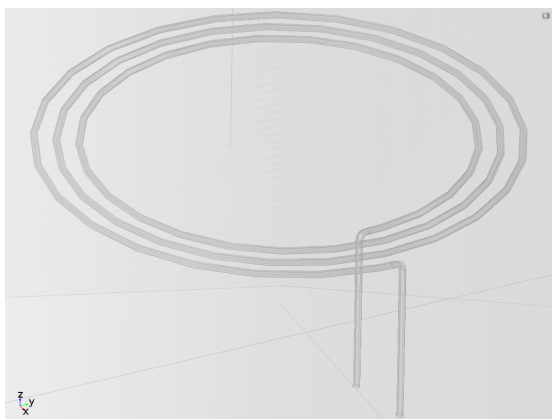

Figure 6. COMSOL-model of the Archimedean spiral coil with N = 3 number of turns, inner radius R_A = 2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm, wire radius of R = 0.5 mm and connection wire length l = 2.2 cm.

In this simulation, the resonance frequency is about 10 MHz lower compared to the first simulation, as shown in figure 7, due to the connection wires increasing the capacitance and inductance.
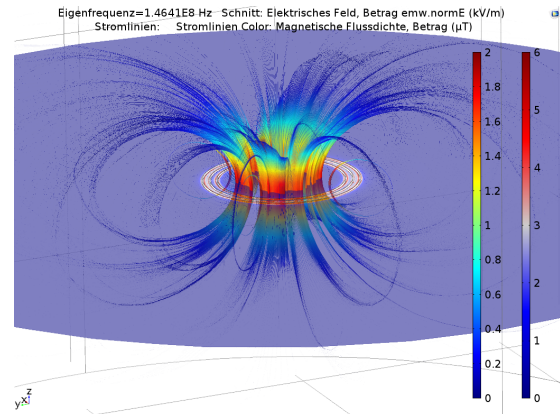

Figure 7. COMSOL-solution of simplified model of an Archimedean spiral coil. The resonance frequency is $f_R \approx 146$ MHz. The left colorbar displays the norm of the magnetic flux density in µT and the right colorbar the electric field intensity in kV/m.

4.3 Simulation of an Archimedean Spiral Coil (3)

Like shown in figure 2 the connection wires are not exactly parallel. In order to obtain a more detailed model, the bending of the inner connecting wire was considered in the next step. Figure 8 shows an example of this. The examined angles are 12°, 15° and 18°.
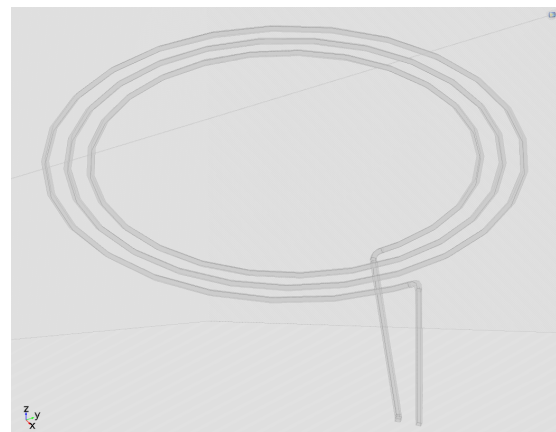

Figure 8. COMSOL-model of an Archimedean spiral coil with N = 3 number of turns, inner radius R_A = 2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm, wire radius of R = 0.5 mm, connection wire length l = 2.2 cm and bending angle of 18°.

The result of the simulation is shown in table 1. A higher bending angle results in lower resonance frequency. This is exactly what was to be expected. If the distance between the wires is decreased, the inductance and the capacitance will rise up. Because of this fact, the resonance frequency decreases.

$\theta/°$	f_R/MHz
12	144.81
15	144.07
18	143.12

Table 1: Solutions for Resonance Frequency. Sweep Parameter is the bending angle θ. Solving time is round about 6min, with 2.4GHz and 8GB

The next step to get a more detailed model should be to include the SMA socket like shown in figure 2 (b). Due to it's fine and detailed structure, this component should be disregarded. The small distances in the geometry and the relative permittivity of the SMA socket's material, affect that the inductance and capacitance will increase furthermore, leading to a decreased resonance frequency. So these influences are therefore not negligible when discussing the simulation results.

4.4 Simulation of an Archimedean Spiral Coil (4)

The last extension of the model, like shown in figure 9, was the inclusion of the cardboard to which the coil is attached. The problem with this model is that the relative permittivity of the cardboard is unknown. Many references say the relative permittivity of paper to be in the range of ε_r = 1.2...4 and the value of permeability was chosen to μ_r = 1. So the best way to get an overview of the influence of the cardboard is a parameter sweep on the relative permittivity.
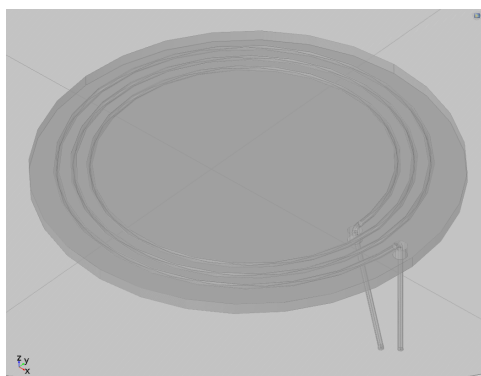

Figure 9. COMSOL-model of an Archimedean spiral coil with N = 3 number of turns, inner radius R_A=2.5 cm, average wire distance d = 3 mm, arc length L = 55.614 cm, wire radius of R = 0.5 mm, connection wire length l = 2.2 cm, bending angle of 18°, thickness of cardboard d = 2 mm and radius of cardboard 3.7 cm.

Hence of its manufacturing, like shown in figure 2, some positions of the Archimedean spiral coil differ from the ideal geometry in terms of distance and symmetry of windings. Due to this, the resonance frequency of the experimental coil may be smaller than simulated. So it seems interesting to perform a parameter sweep on the radius of the wire and the average distance between the windings.

The results of simulation are shown in table 2. The higher the relative permittivity of the paper plate,

ε_r	R/mm	d/mm	f_R/MHz
2	0.4	2.8	137.7
2	0.4	3	139.1
2	0.4	3.2	140.22
2	0.5	2.8	134.38
2	0.5	3	136.13
2	0.5	3.2	137.41
2	0.6	2.8	130.87
2	0.6	3	132.87
2	0.6	3.2	134.31
3	0.4	2.8	131.77
3	0.4	3	133.1
3	0.4	3.2	134.16
3	0.5	2.8	129.22
3	0.5	3	130.83
3	0.5	3.2	132.09
3	0.6	2.8	126.35
3	0.6	3	128.16
3	0.6	3.2	129.48
4	0.4	2.8	127.55
4	0.4	3	128.84
4	0.4	3.2	129.78
4	0.5	2.8	125.52
4	0.5	3	127.05
4	0.5	3.2	128.21
4	0.6	2.8	123.04
4	0.6	3	124.72
4	0.6	3.2	125.93

Table 2: Parameter Sweep Solutions for Resonance Frequency. Sweep Parameters are the relativ permittivity of the cardboard ε_r, the radius of the wire R and the average wire distance d. Solving time is round about 1h 23min, with 2.4GHz and 8GB RAM.

the lower the resonance frequency. Compared to the previous simulations the maximum reduction is about 20MHz and the minimum reduction about 2MHz. In conclusion the real value can be found between the maximum and minimum.

5. Discussions and Conclusions

COMSOL's RF module offers a simple but efficient method for determining the resonance frequencies of coils. For the designing process of coils, it is very helpful to estimate the resonance frequency. Because the models are not arbitrarily accurate, the simulated resonance frequency will always deviate from the measurement.

If the SMA socket is neglected, it can be said that the simulation results are well suited to the measurement. By comparing the influence of the connection lines, it could be estimated that the SMA socket reduces the resonance frequency by 10 to 20MHz. Furthermore, the simulation lacks the influence of the skin effect, because the inner area of the wire is ignored. In this case, the influence on the resonance frequency cannot be predicted. However, in the case of high conductivity, the simulation should provide a good approximation. If the relative permittivity of the board were known, the influences of the individual parts could be better distinguished. The COSMOL simulation had many problems to deal with. If the cardboard was inserted directly under the coil during its modeling in COMSOL, problems with mesh generation appeared. To solve this problem, the cardboard was placed a little deeper under the coil. Because of the modeling of the bended connection wires, there were many very small geometry objects like shown in figure 10. Therefore, for the design of the mesh, it was necessary to create size objects for each part of the geometry and manually set the minimum and maximum element size. A dilemma arises from the fact that the finer the FEM grid is, the longer the computing time becomes. Therefore, a well-balanced compromise must be made in terms of accuracy and calculation time. As can be seen in Figure 10, the compromise reached here is that the actual round coil wire is rather rectangular. This should also cause a certain influence on the simulated resonance frequency.
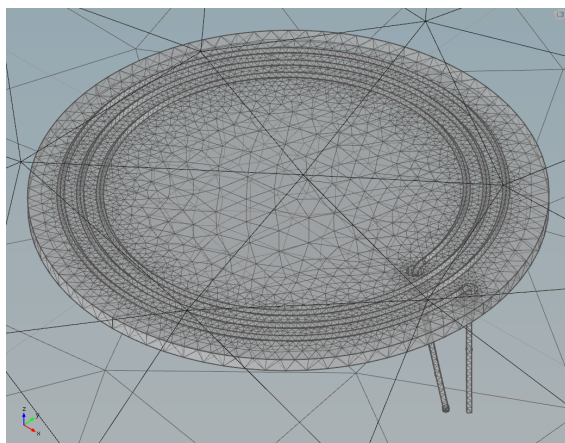

Figure 10. COMSOL model of an Archimedean spiral coil with FEM grid. The lower part of the inner connecting wire consists of very small geometric objects. It can be seen that due to the fineness of the grid the actually round coil wire is rather angular.

If the cylindrical volume for the limit of the model is too small, the resonance frequency will rise up, because at the boundary of the geometry the field is not exactly converged to zero. So in this case, too

many field components are neglected. For the designing of the coil model in COMSOL it is firstly necessary to create parameter curves, afterwards the sweep function is used to create the circular surface around the curve. If only the Archimedean spiral is built as parameter curve and sweep function, and the other parts e.g. the connection wires are built with cylinder, COMSOL cannot unite the geometry parts. Physically incorrect results may occur in the COMSOL study on eigenfrequency as illustrated in Figure 11. In the case of figure 11, the eigenfrequency apparently results from the structure of the FEM grid. In order to avoid this problem, on COMSOL *study* should set the *number of desired eigenfrequency* on two. Thus this it should be guaranteed that the simulation result contains the correct resonance frequency.
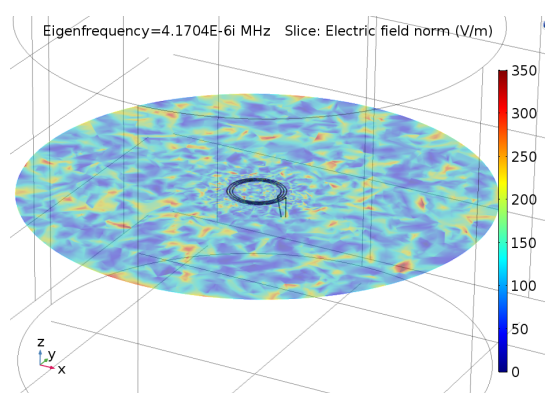

Figure 11. COMSOL solution of physically wrong Eigenfrequency. The incorrect eigenfrequency apparently results from the structure of the FEM grid.

6. References

1. J.W.Hooker, V.Ramaswamy, R.K.Arora, A.S.Edison andW.W.Brey, "Effects of dielectric substrates and ground planes on resonance frequency of archimedean spirals," *IEEE Transactions on Applied Superconductivity*, vol. 26, no. 3, pp. 1–4, 2016.
2. S. Eroglu, B. Gimi, B. Roman, G. Friedman, and R. L. Magin, "Nmr spiral surface microcoils: design, fabrication, and imaging," *Concepts in Magnetic Resonance Part B: Magnetic Resonance Engineering*, vol. 17, no. 1, pp. 1–10, 2003.
3. O. Georg, *Elektromagnetische Felder und Netzwerke: Anwendungen in Mathcad und PSpice*. Springer-Verlag, 2013.
4. O. Georg, *Elektromagnetische Wellen: Grundlagen und durchgerechnete Beispiele, Springer Verlag 2013*