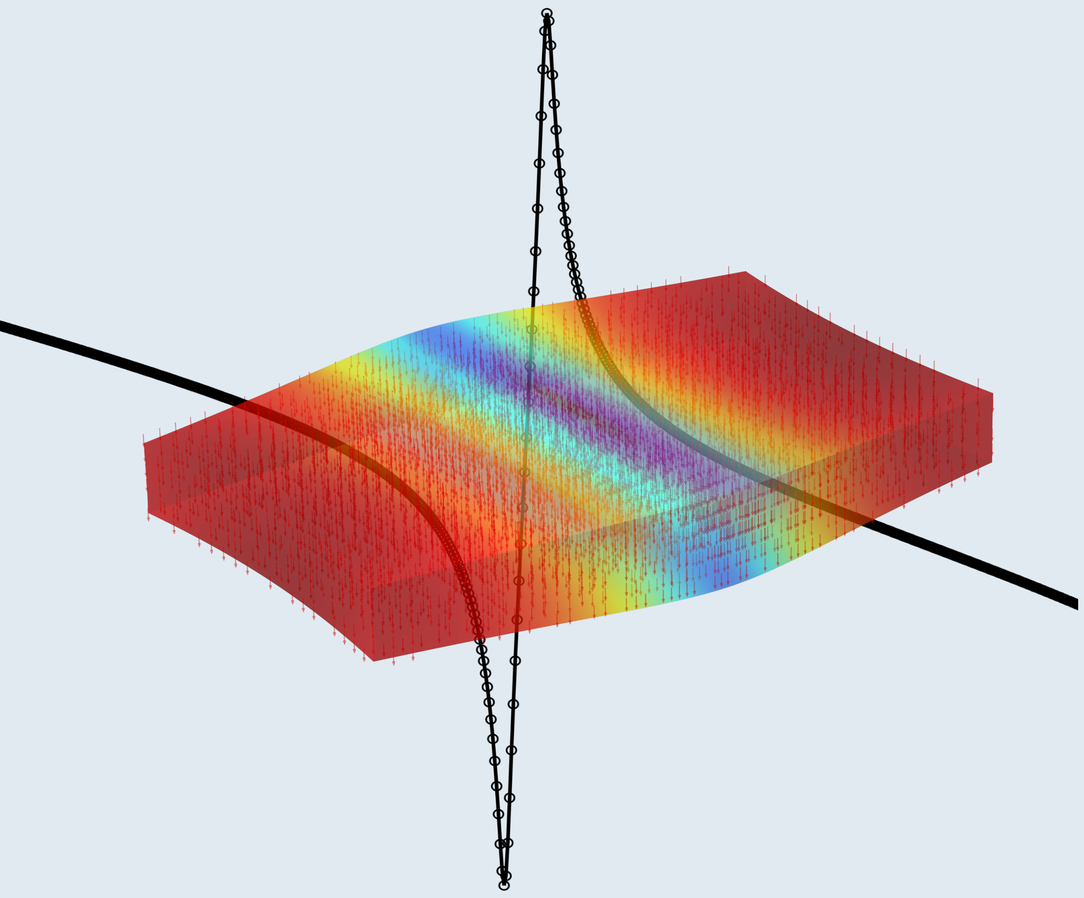

Sensitivity Analysis of Piezoelectric Material Parameters Using Sobol Indices

The applicability of the method of Sobol to the non-smooth, resonant behavior of a vibrational eigenmode of a piezoelectric element is examined.

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Abstract

The goal is to quantify the sensitivity of a piezo element's vibrational eigenmode upon variation of the piezoelectric material parameters. The calculation of Sobol indices is a statistical method, which means that many randomly generated parameter samples are required (Ref. 2). These are taken from a Latin hypercube sampling (LHS). For each such sampling point a frequency-domain study is performed, resulting in an electrical impedance curve, see Fig. 1. From this curve the resonance position are extracted, by means of which the Sobol indices are calculated.

It is found that the Sobol index highest in magnitude, which ought to represent the material parameter with the largest sensitivity, agrees with the one known from literature to be most influential for the eigenmode properties of the considered block-shaped sample (Ref. 1).

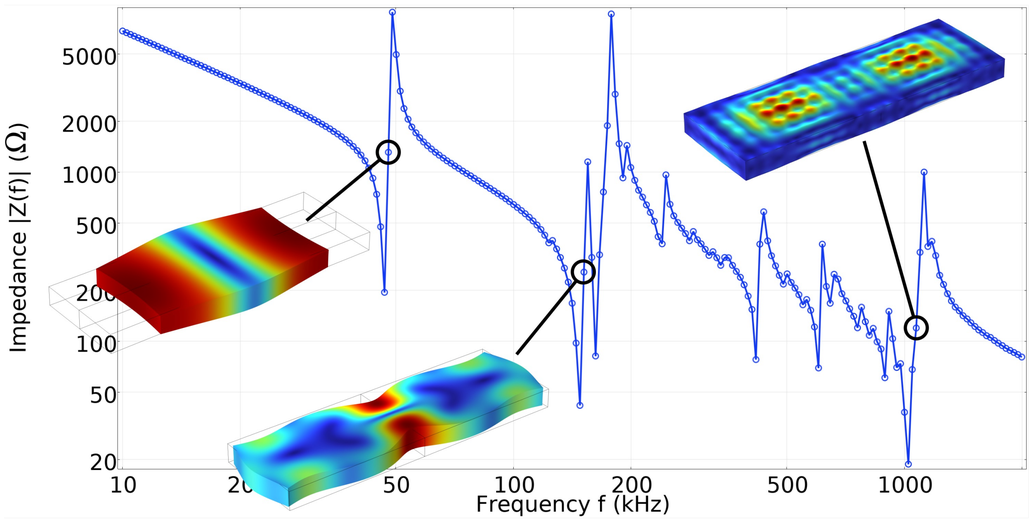

FIGURE 1. Electrical impedance magnitude. Three different eigen-modes are shown, from which the leftmost is used.

Methodology

A hybrid computational approach by means of the LiveLink™ *for* MATLAB® feature is chosen. The LHS process, the COMSOL® model creation, the extraction of the resonance frequencies, and the calculation of the Sobol indices is performed in MATLAB®. COMSOL® is used for the computation of the electrical impedance of the considered block-shaped piezoceramic test specimen. To this end, a frequency-domain study is set up, combined with a parametric sweep with the LHS parameter combinations as input. A batch sweep is employed to speed up the computation by parallel model execution (Ref. 3).

Results

Figure 2 shows the evolution of the indices with the sample size. While the first order Sobol indices vary quite substantially, they already distinguishing the influential parameters from the remaining ones. The total Sobol indices support the trends seen in the first order index. As they show not much variation upon changing sample size, the total Sobol index can be quite useful to assess the importance of the model parameters more quickly.

Further studies with larger sample sizes for the first order Sobol indices are necessary. In this context, the Cluster Sweep functionality of COMSOL® could be a promising approach. Alternatively, one could employ a computationally cheaper eigenmode study.

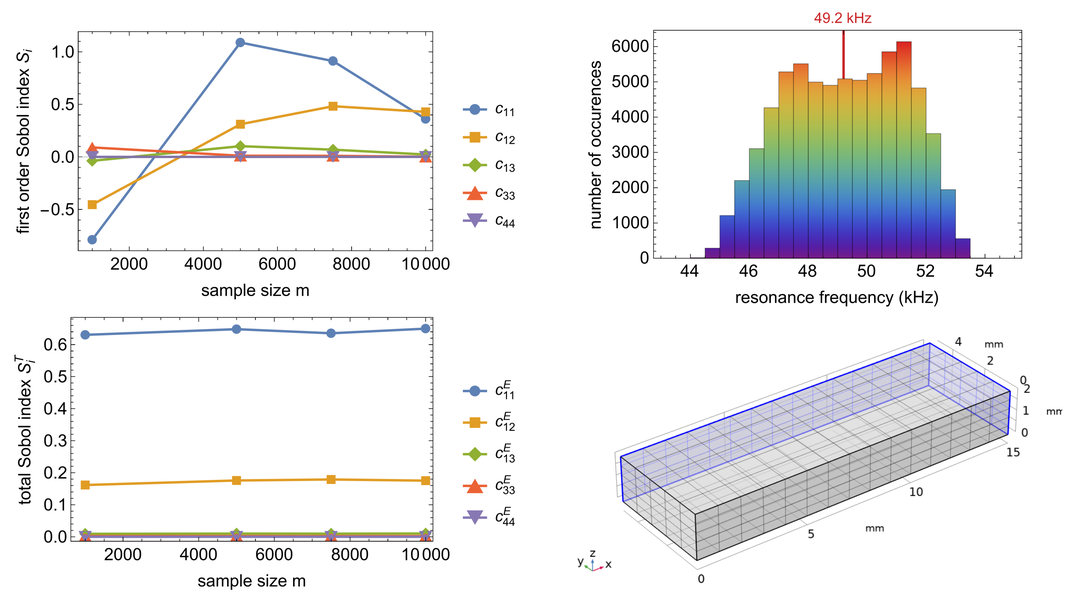

FIGURE 2. Left: Evolution of Sobol indices with the sample size. Right: Distribution of computed resonance positions; model geometry and mesh (highlighted: symmetry planes).

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