Wave Carpet Simulation Using Coupled Hydro-Elastic FEMLAB Model

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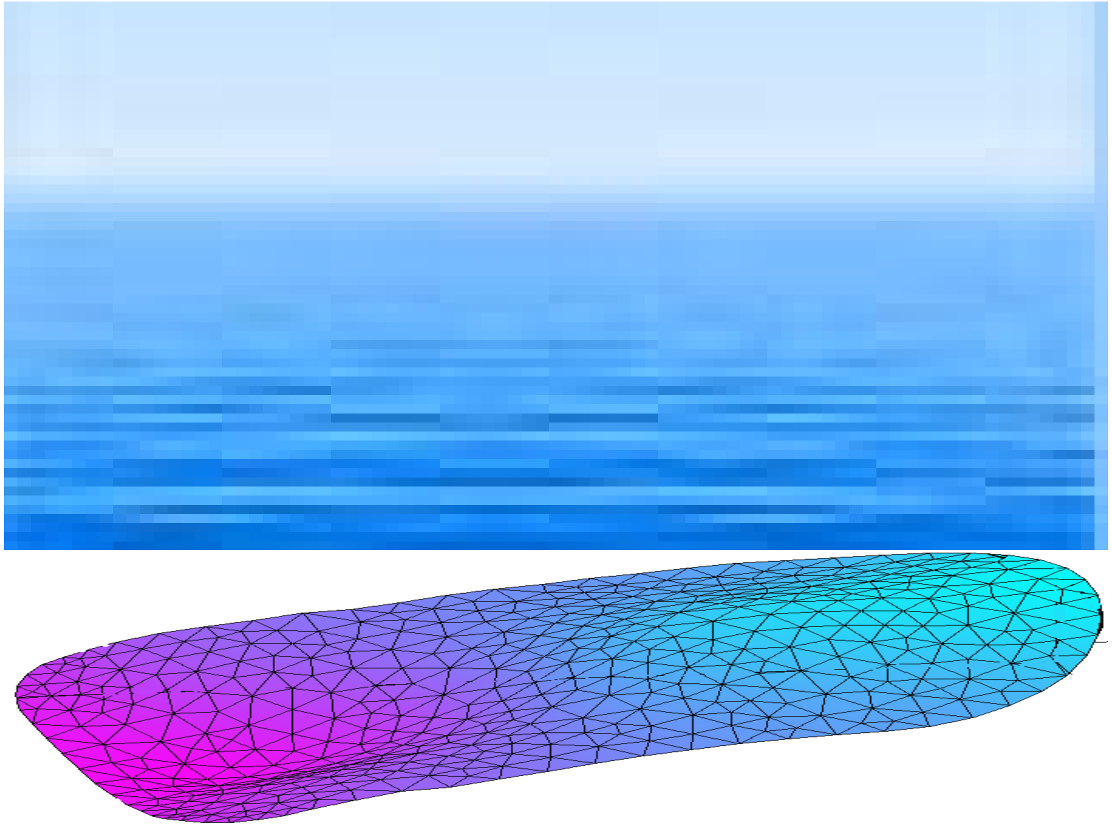

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Comments and Acknowledgements I

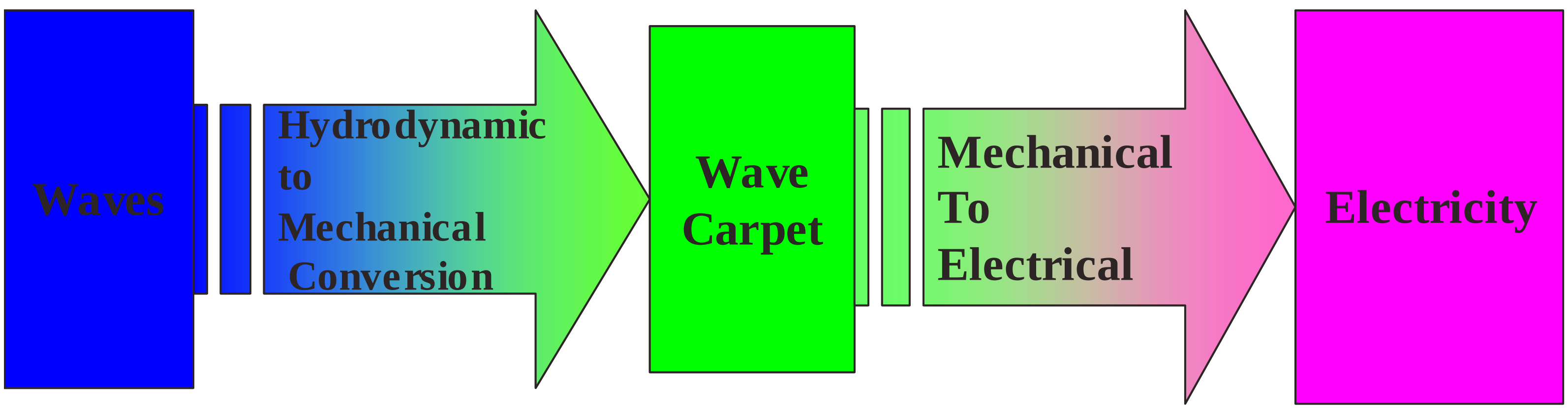
- This work was conducted under project “Wave Carpet” and sponsored by DoD, SBIR NAVY02-051.
- This study was applied for novel method of the wave energy extraction and was presented at the conference in San Diego A.I. *Ibragimov. A, Koola P*. The dynamics of Wave Carpet- A novel deep water wave energy design, OCEANS 2003, MTS/IEEE conference, San Diego, California, proceedings, 2288-2293.
- Our special thanks to Dr. Richard Mayer, President KBSI for his enthusiasm, encouragement and support. Part of this work was sponsored by the Office of Naval Research under SBIR contract No. N00014-02-M-0147 titled “Wave Carpet”.
- We also want acknowledge input of Nils Malm from COMSOL support group, especially during the initial phase of the development of FEMLAB model using advanced features of the package.

Comments and Acknowledgements II

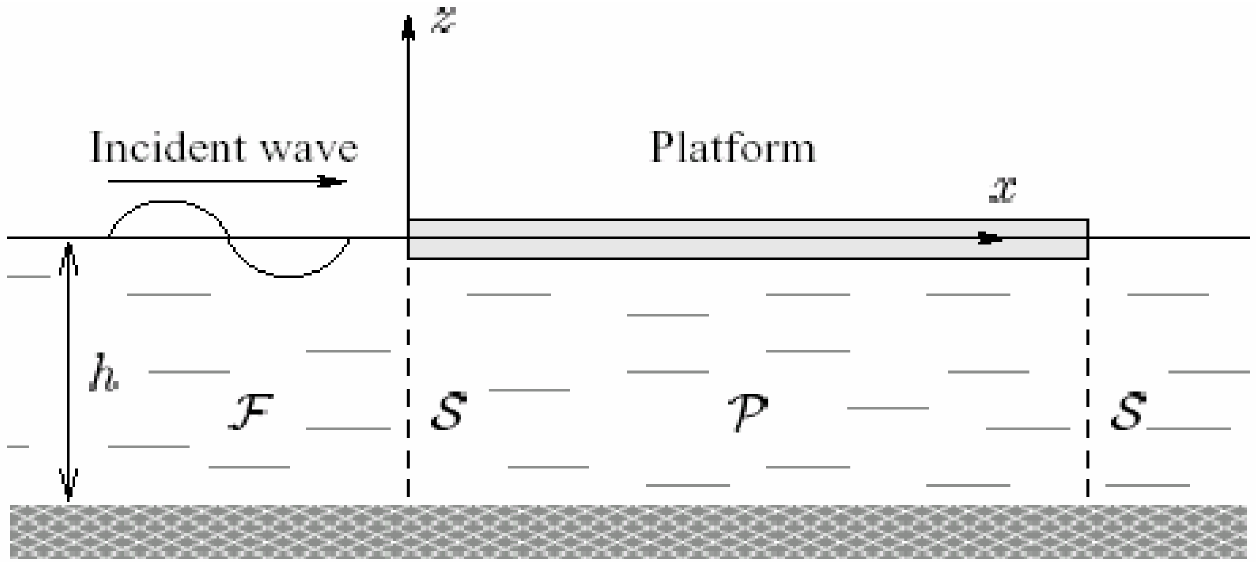
Wave Carpet was my First Experience using FEMLAB in 2002, but it was only beginning below is list of Industrial, research, and educational projects, in which COMSOL was applied as a major tool

- Advanced NDI Techniques (NDI) , KBSI
- Ship Hull Corrosion Protection Using Data Mining Techniques, KBSI
- Multi-Sensor Fusion and Mitigation of Crevice Corrosion, KBSI
- Leak Inspection, Quantification and Detection System, KBSI
- Multi-Sensor Features Fusion, KBSI
- Corrosion Image Enhancement Using Concept of Nonlinear Diffusive Reaction Nonlinear Equations, KBSI
- Novel Mathematical Framework in Reservoir Engineering for Productivity Index of the Well (Texas A&M).
- Mathematical Frame Work for Atherosclerosis Initiation (Texas A&M, Texas Tech)
- Traveling Wave Phenomena Study for generalized KPP type reaction diffusion system (Texas A&M, Texas Tech)
- Special Topic Course for Graduate Student: Special Issues in Applied Mathematics and Applications to Cell Biology and Medicine, 12 individual projects (Texas Tech)

Wave to Wire System Chain



Definition Sketch of the Problem



Assumptions about liquid and liquid motion:

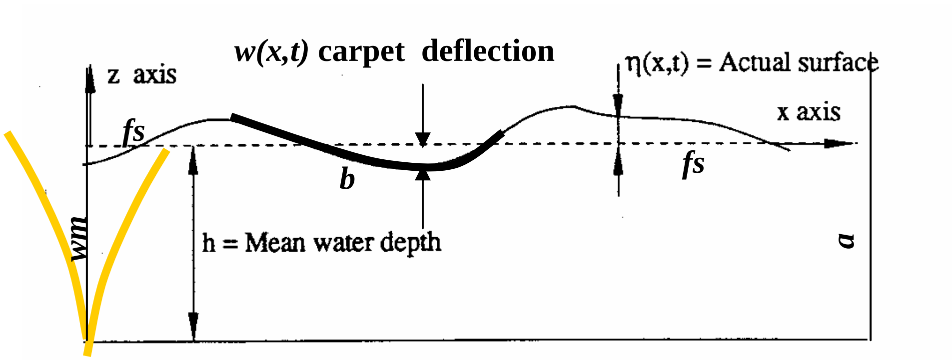
- *Incompressible:* $\rho = const$
- *Ideal* $\mu = 0$
- *Irrotational* $curl\ \vec{v} = 0$

$$\exists \quad \Phi : \vec{v} = grad\ \Phi \qquad (1)$$

$$\Delta\Phi = 0 \qquad (2)$$

1. *Andrianov, Hermans A* VLFP on Infinite, Finite and Shallow Water;
2. *Khabakhpasheva T, Korobkin A*, Exact Solution of Floating Elastic Plate Problem,
Both paper presented at17-th International workshop on water waves and floating bodies, Cambridge,UK, April 14 - 17, 2002.http://www.wsatkins.com/17iwwwfb/proceedings

Assumptions about liquid surface(Linearlized Statement)



From Euler equation and Irrotational condition

$$grad\ \Phi_t + \frac{1}{2} grad(u^2 + v^2 + w^2) = -grad(\frac{P}{\rho} + gz)$$

$$\Phi_t + \frac{1}{2}(grad\ \Phi)^2 + \frac{P}{\rho} + gz = 0$$

Bernoulli

Assume $z = \eta(x,y,t)$ is an equation for water surface

Let:

n>1

$$\left|\frac{\partial^n \eta(x,y,t)}{\partial t^n}\right| << \left|\frac{\partial \eta(x,y,t)}{\partial t}\right|$$

$$(grad\ \Phi)^2 << \Phi_t$$

$$\begin{cases} \dfrac{\partial \Phi}{\partial t} + g\eta = 0 & (3) \\ \dfrac{\partial \Phi}{\partial z} = \dfrac{\partial \eta}{\partial t} & (3') \end{cases}$$

Stoker J.J. Water Waves, The Mathematical Theory with Application

Equations for Free Surface and Plate

$$\Phi_z = -(1/g)\frac{\partial^2 \Phi}{\partial t^2} \quad when \quad z=0 \quad and \quad (x,y)\in F$$ (4) Free surface boundary condition

$$m\frac{\partial^2 w(x,y,t)}{\partial t^2} + k\frac{\partial w(x,y,t)}{\partial t} + D\Delta^2 w = P(x,y,t), \quad z=0, \quad (x,y)\in P$$ (5) Timoshenko-Euler beam equation for elastic Plate

$$P = -\rho\frac{\partial \Phi}{\partial t} - \rho g w$$ (6) Equation for hydrodynamic pressure along the beam which given by the linearized Bernoulli's equation

k- Damping coefficient
w- Plate deflection
m- Mass of unit area
D- Equivalent flexural rigidity

$$\left(\frac{m}{\rho g}\frac{\partial^2}{\partial t^2} + \frac{k}{\rho g}\frac{\partial}{\partial t} + D\Delta^2 + 1 + \frac{k}{\rho g}\right)\Phi_z - \frac{1}{g}\frac{\partial^2 \Phi(x,y,0)}{\partial t^2} = 0$$ (7)

$$\frac{\partial^2 w}{\partial n^2} + \nu\frac{\partial^2 w}{\partial \tau^2} = 0, \quad and$$
$$\frac{\partial^3 w}{\partial n^3} + (2-\nu)\frac{\partial^3 w}{\partial \tau^2 \partial n} = 0$$
$$on \quad the \quad edges \quad of\ the\ plate$$ (8) Free of moment and shear stress conditions

Initial and Boundary Condition

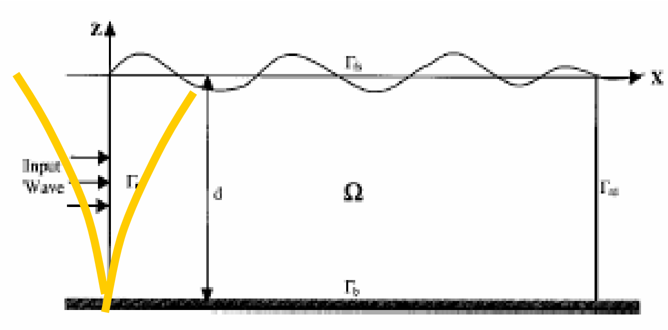


Fig. 1 Definition sketch

- **a)** By prescribing wave maker motion on the upstream boundary
- **b)** By specifying wave behavior on controlled boundary

$$\left.\frac{\partial \Phi}{\partial x}\right|_{x=0} = f_0(y,z,t) \qquad (9)$$

Wave maker condition

$$\frac{\partial \Phi}{\partial x} = -\frac{1}{c}\frac{\partial \Phi}{\partial t} \qquad (10)$$

Zomerfeld radiation condition

$$\left.\frac{\partial \Phi}{\partial y}\right|_{y=0,L_y} = 0 \qquad (11)$$

Non-flow condition in *y* direction

$$\Phi(x,y,z,0) = h(x,y,z) \qquad (12)$$

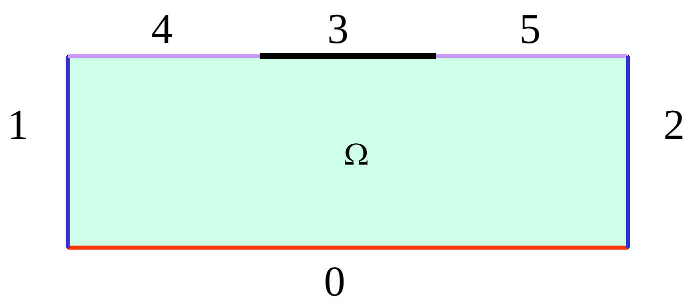
Initial condition

$$\left.\frac{\partial \Phi}{\partial z}\right|_{z=-h} = 0 \qquad (13)$$

Non-flow condition in the bottom of the see/ocean

Baudic S.F. Williams A.N., Kareem A., A two-Dimensional Numerical Wave Flume, Transaction of AME, Vol. 123, May, 70-74, 2001.
Orlanski, A simple Boundary for Unbounded Hyperbolic Flows, J.Comp. Phys., 21, pp 251-269, 1976.
[Bengquist B, Majda A., Absorbing Boundary Conditions for Numerical Simulation of Waves Mathematics of Computation, V. 31, N0 139, July 1977.

Final Formulation of Initial boundary Problem



$$\nabla^2 u(x,y,t) = 0 \quad in \quad domain \quad \Omega: \; -d < y < 0, \quad -L < x < L,$$ In domain (14)

$$u_z = \frac{\partial u}{\partial z} = 0$$ On the boundary (0) (15)

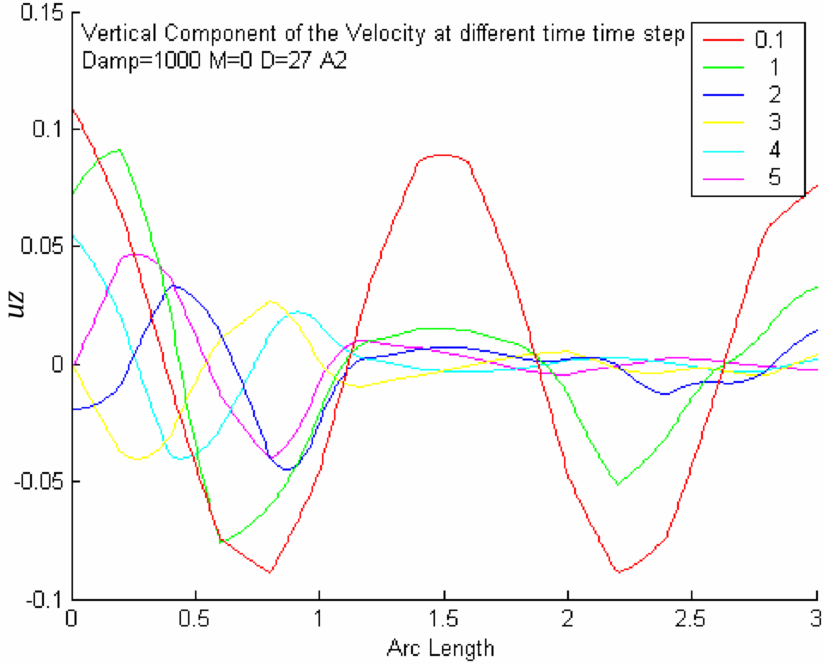
$$u_y = -(1/g)\frac{\partial^2 u}{\partial t^2} \quad when \quad y = 0 \quad and \quad -L < x < -L_0; L_0 < x < L$$ On the boundary (4) and (5) (16)

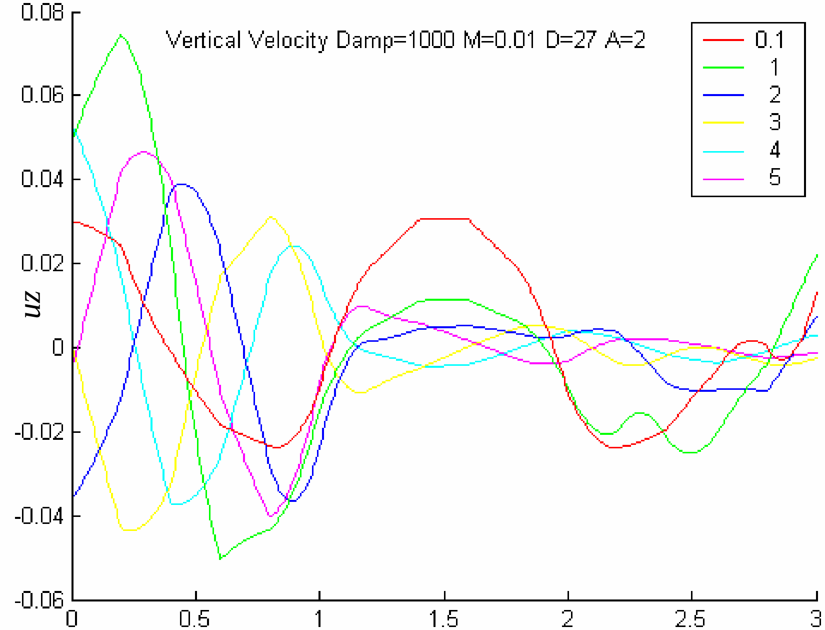
$$m\frac{\partial^2 u_y(x,y,0,t)}{\partial t^2} + c\frac{\partial u_y(x,y,0,t)}{\partial t} + D\frac{\partial^4 u_y}{\partial x^4} + \rho g u_y - \rho\frac{\partial^2 u(x,y,0,t)}{\partial t^2} = 0$$ On the boundary (3) (17)

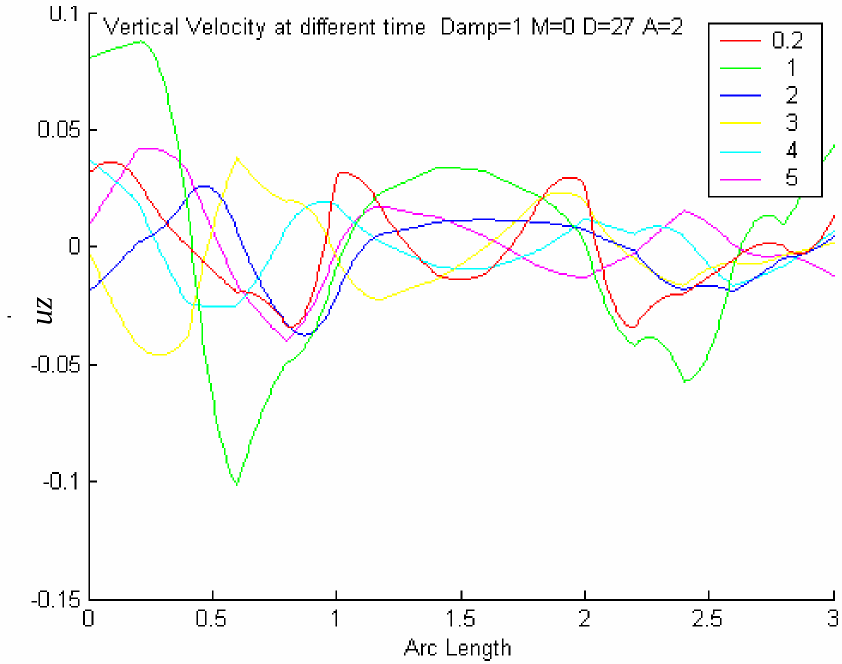
$$\frac{\partial u}{\partial x} = -\frac{1}{a}\frac{\partial u}{\partial t}$$ On the boundary (2) (18)

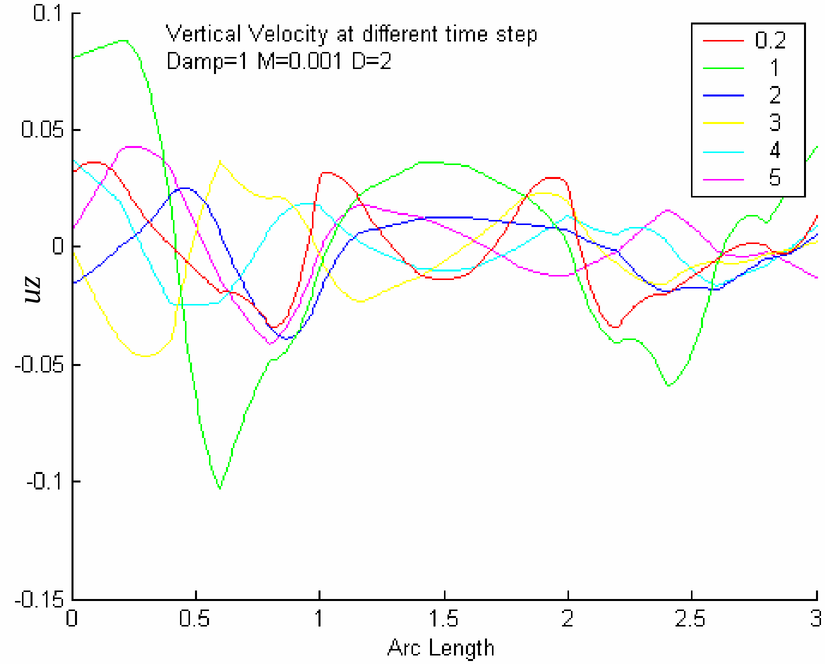
$$\frac{\partial u}{\partial x} = A\cos(\omega t)\exp(-a_0 y)$$ On the boundary (1) (19)

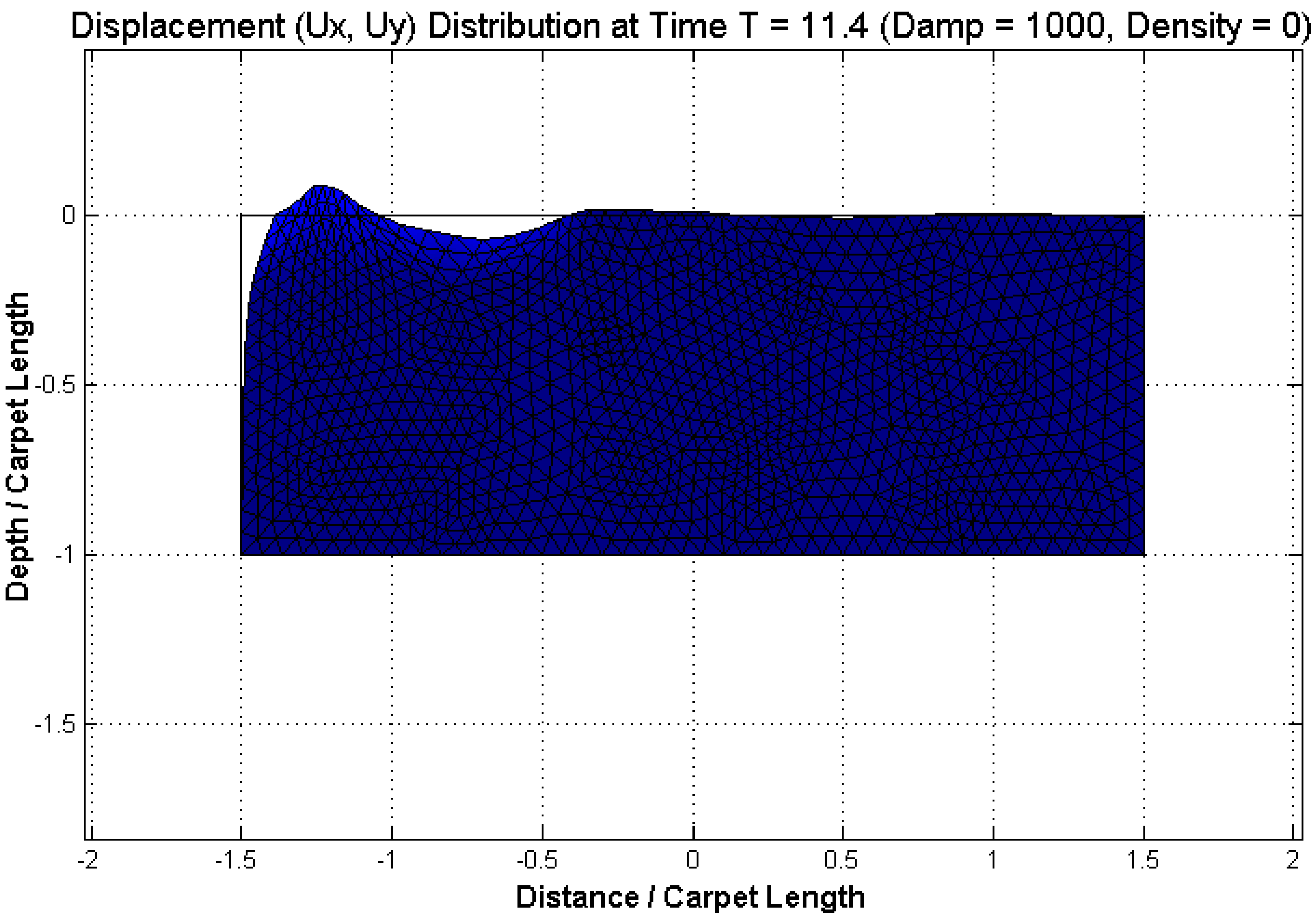
$$u(x,y,0) = h(x,y)$$ At initial time in domain (20)


Vertical Component of the Velocity at different time time step
Damp=1000 M=0 D=27 A2
0.1
1
2
3
4
5
uz
0.15
0.1
0.05
0
-0.05
-0.1
0
0.5
1
1.5
2
2.5
3
Arc Length

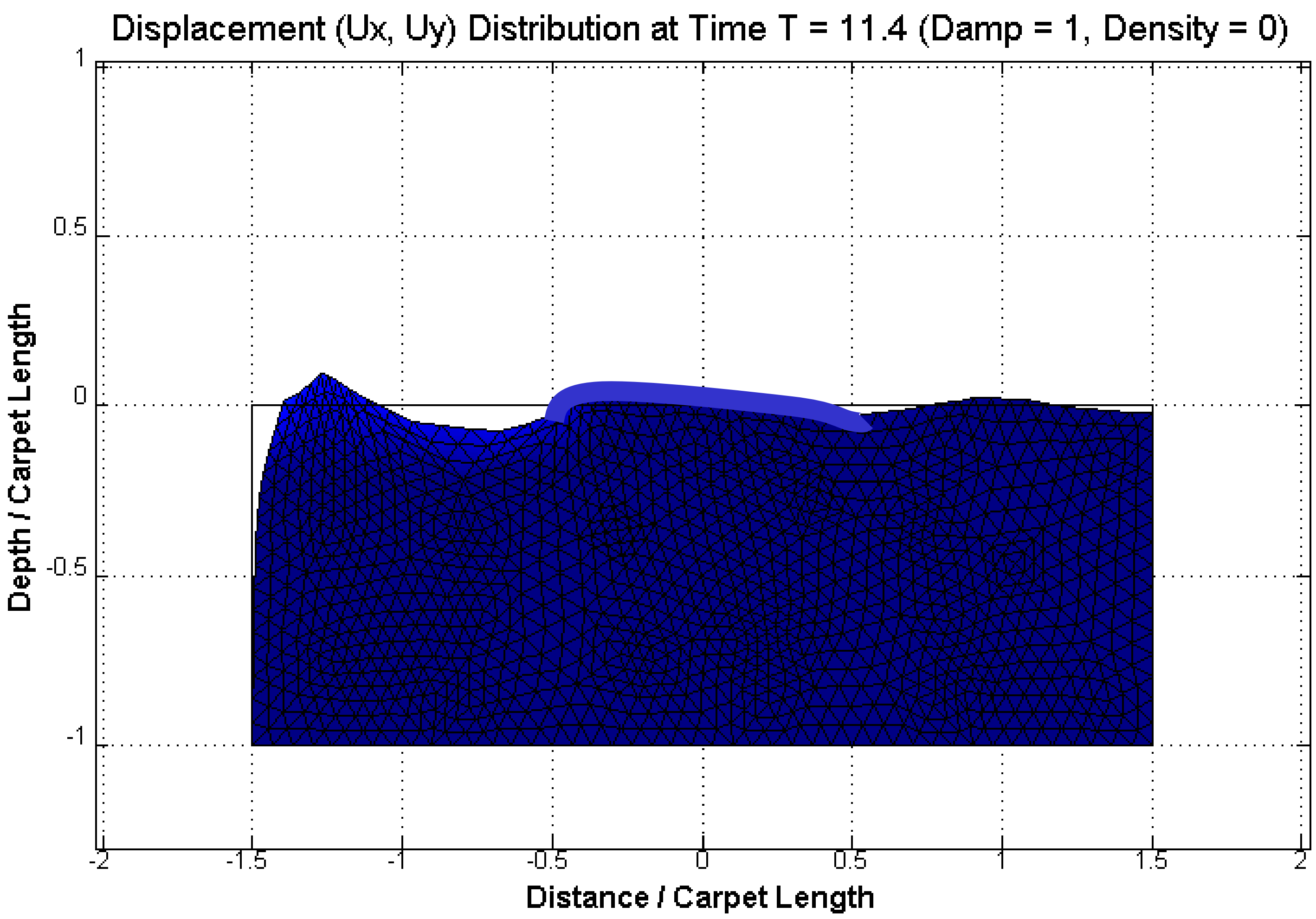

Vertical Velocity Damp=1000 M=0.01 D=27 A=2
0.1
1
2
3
4
5
uz
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
0
0.5
1
1.5
2
2.5
3


Vertical Velocity at different time Damp=1 M=0 D=27 A=2
0.2
1
2
3
4
5
uz
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.5
1
1.5
2
2.5
3
Arc Length


Vertical Velocity at different time step
Damp=1 M=0.001 D=2
0.2
1
2
3
4
5
uz
0.1
0.05
0
-0.05
-0.1
-0.15
0
0.5
1
1.5
2
2.5
3
Arc Length

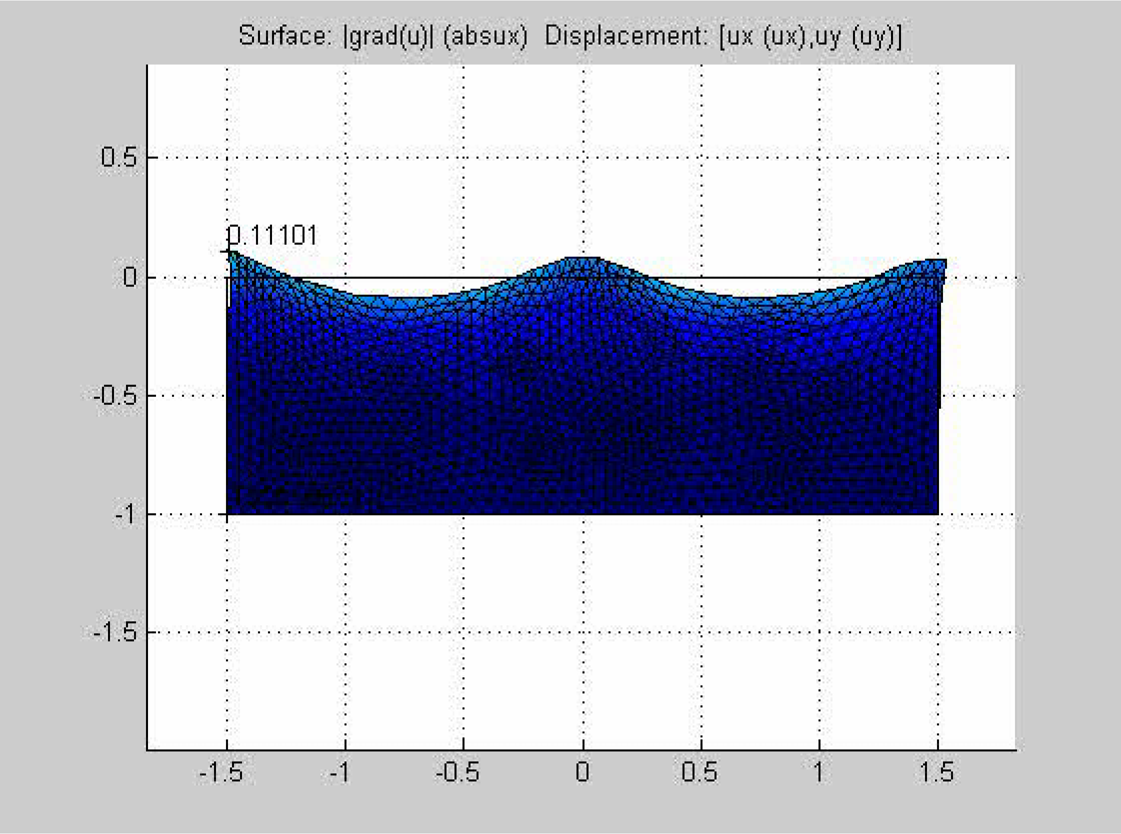


Displacement (Ux, Uy) Distribution at Time T = 11.4 (Damp = 1000, Density = 0)
0
-0.5
-1
-1.5
Depth / Carpet Length
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Distance / Carpet Length

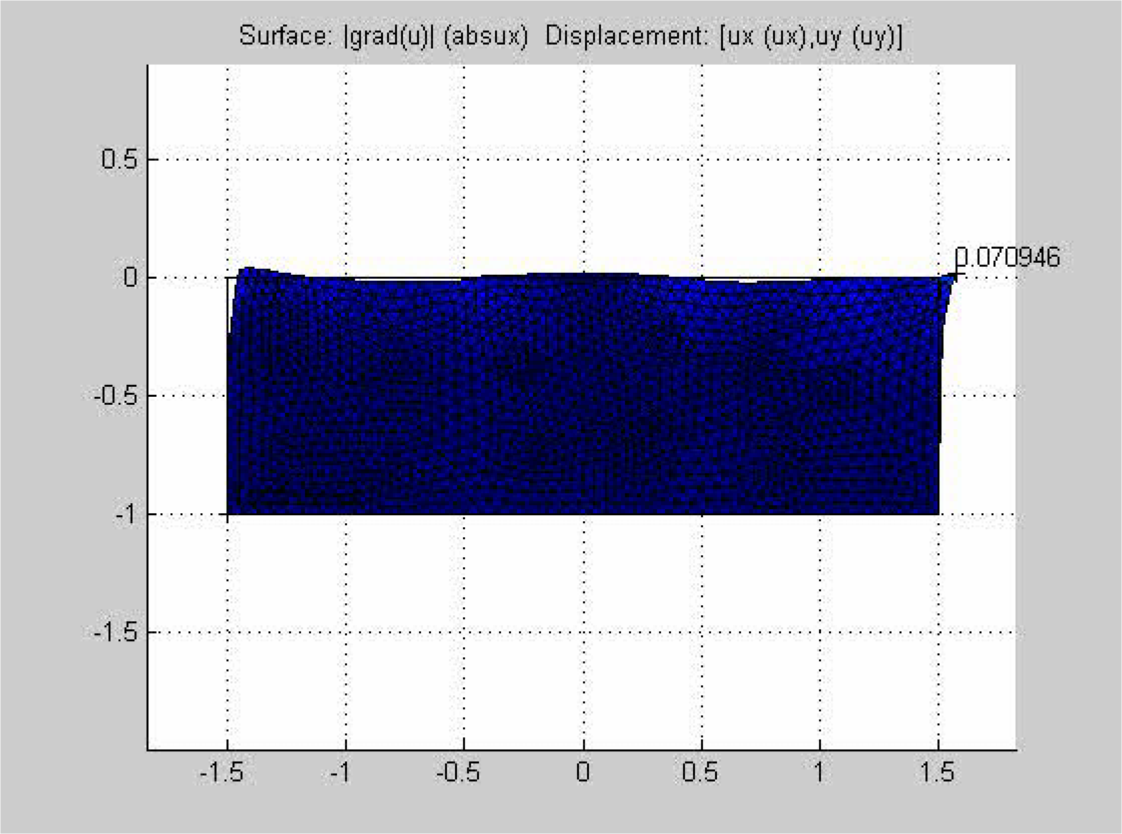


Displacement (Ux, Uy) Distribution at Time T = 11.4 (Damp = 1, Density = 0)
1
0.5
0
-0.5
-1
Depth / Carpet Length
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
Distance / Carpet Length

Simulation of Wave Carpet



Simulation of Wave Carpet



Conclusions

- This paper presents a new deep-water wave energy device the Wave Carpet. Based on a review of past work we discuss the main issues that would be critical for such a deep-water rapidly re-deployable design. We then conceptually introduce the wave carpet as a feasible solution.
- We then propose a theory to model the carpet using the Numerical Wave Tank Concept and formulate the coupled hydro-elastic time-domain solution for wave generation and carpet motion in a tank. We then solve this problem using essential features and advances in FEMLAB in two dimensions and present typical results.

Afterwards, What Comsol Give to Applied Mathematician: Personal Point of View?

- New vision: ability look on problems in different fields of science and engineering?
- Test hypotheses (not only your own) and correct models of the processes
- Make work ,in fact, more fundamental from mathematical and applied points of view