COMSOL MULTIPHYSICS®

Coupled Flow Laws

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Introduction

Understanding what happens during the transition from slow flow in porous media to fast flow in a channel is critical in many environmental cases and applied problems This type of flow appears near rivers, estuaries, wellbores, caverns, and lava tubes, to name a few examples.

Traditionally, the quantitative assessment of transitioning flows has been the domain of those with time and tools to work out their own code because it requires switching between mathematical expressions for different flow laws. Darcy's law describes slow flow at a distance from the channel; the Navier-Stokes equations govern free or open-channel flows; and in between, where the fluid moves in porous media but shear is nonnegligible, the Brinkman or Forcheimer equations apply. This example demonstrates how to model such a transition using predefined equations in the Earth Science Module.

The following model examines transitioning flow by zooming in on oil movement to and within a perforated well. The analysis begins by coupling Darcy's law and the Brinkman equations to represent flow in porous media that quickens toward a perforation in the well casing. Next, it examines fluid movement into and within the well by coupling the Navier-Stokes equations to the Darcy-Brinkman model. Albeit counterintuitive, time-dependent Brinkman and Navier-Stokes are well known to be relatively easy to solve. This model instead analyzes a steady-state system.

By giving you the ability to link flow laws, modify predefined equations, and even freely define your own governing equations, COMSOL Multiphysics sees abundant use in analyzing conventional, coupled, and nonconventional flows (see Ref. 1 and Ref. 2).

Model Definition

This Darcy-Brinkman example begins by overviewing the model setup and continues with the equations and the boundary conditions used in the analysis. Implementation details follow the mathematical background. The model triggers weak variables to implement the coupling between Darcy's law and the Brinkman equations. Finally this discussion reviews results and outlines the mechanics for building the model. The next example (Transitional Flow: Darcy-Brinkman–Navier-Stokes on page 69) adds the Navier-Stokes equations directly on top of the Darcy–Brinkman model file.

First examine a general description of the Darcy–Brinkman model depicted in Figure 1. Oil moves through a thin porous layer towards a perforation to a well. The fluid flow follows Darcy's law in the far field (that is, 1 m < x < 4 m) and the Brinkman equations near the well opening (between 0.1 m < x < 1 m). The layer is 0.875 m thick and bounded above and below by impermeable materials that confine the permeable reservoir layer. For simplicity, assume the layer has homogeneous and isotropic hydraulic properties, and the fluid has constant density and viscosity. You know the flux of fluid at the inlet and the pressure at the perforations. The flow field is steady state.

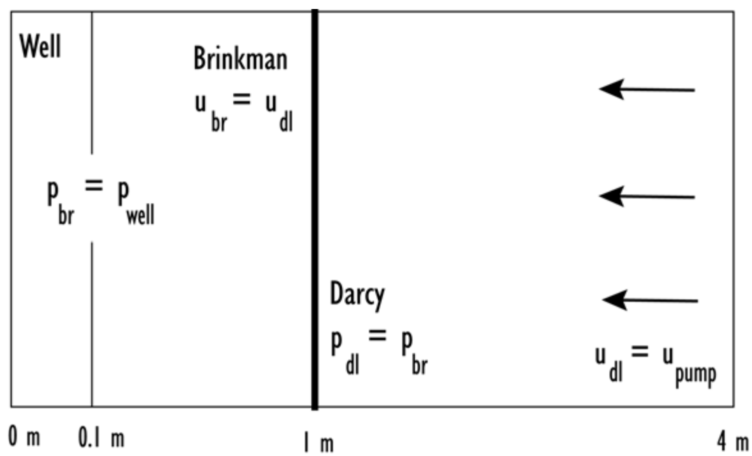

Figure 1: Model geometry showing boundary conditions for coupling Darcy's law (1 m < r < 4 m) and the Brinkman equations (0.1 m < r < 1 m).

DARCY'S LAW

Darcy's law describes fluid flow driven by gradients in pressure and elevation potential. The dependent variable in Darcy's law is pressure, p. The flows are slow enough that velocity head is negligible. For a steady state, the governing equation is

$$\nabla \cdot \left[-\frac{\kappa}{\eta}\nabla(p_{\mathrm{dl}} + \rho_{\mathrm{f}} g D)\right] = Q_s$$

In this equation, κ denotes the permeability (m^2), η is the dynamic viscosity (kg/(m·s)), ρ_{f} gives the fluid density (kg/m^3), and g the acceleration of gravity (m/s^2). Further, D is the coordinate for vertical elevation (m), and Q_{s} is the volumetric flow rate per unit volume of reservoir for a fluid source (1/s). You set D to zero in this problem because elevation potential is negligible given that the flow field is very thin. Because this model deals with multiple flow laws, this equation appends the "dl" subscript to p to denote the Darcy's law equation.

With a steady state, flow into the reservoir study area must equal the pumping rate. The Darcy velocity gives the inlet condition as

$$\mathbf{u}_{\mathrm{dl}} = -\frac{\kappa}{\eta}\nabla p_{\mathrm{dl}} = \frac{W}{2\pi r_{\mathrm{res}} b}$$

where W is the volumetric pumping rate for the perforated interval (m^3/s), r_{res} equals the reservoir radius (m), and b is the reservoir thickness (m).

For a continuous solution across the interface between the zones of Darcy and Brinkman flow, the pressure and velocities from Darcy's law must equal the pressure and velocities from the Brinkman equations. Because a Neumann statement on flux already defines the inlet boundary, use the following constraint on pressure for the Darcy–Brinkman interface:

$$p_{\mathrm{dl}} = p_{\mathrm{br}}$$

In this equation, the subscript "br" denotes the Brinkman equations. This expression is a Dirichlet boundary statement.

With no flow through the confining units that overlie and underlie the permeable reservoir zone, the boundary conditions for Darcy's law are

$$\mathbf{n}\cdot\left(\frac{\kappa}{\eta}(\nabla p_{\mathrm{dl}})\right) = -\frac{W}{2\pi r_{\mathrm{res}} b} \qquad \partial\Omega \quad \text{inlet}$$

$$p_{\mathrm{dl}} = p_{\mathrm{br}} \qquad \partial\Omega \quad \text{Darcy-Brinkman interface}$$

$$\mathbf{n}\cdot\left(\frac{\kappa}{\eta}(\nabla p_{\mathrm{dl}})\right) = 0 \qquad \partial\Omega \quad \text{Confining layers}$$

where $\mathbf{n}$ is the unit vector normal to the boundary.

BRINKMAN EQUATIONS

The Brinkman equations describe fluid flow in porous media where velocities are high enough that momentum transport by shear stress is important. Brinkman problems combine a momentum balance in the r and z directions with the continuity equation, giving dependent variables of directional velocities u and v as well as the pressure p. The Brinkman equations for steady state flow are

$$\left(-\nabla\cdot\frac{\eta}{\varepsilon}(\nabla\mathbf{u}_{\mathrm{br}}+(\nabla\mathbf{u}_{\mathrm{br}})^T)\right)-\left(\frac{\eta}{k}\mathbf{u}_{\mathrm{br}}+\nabla p_{\mathrm{br}}-\mathbf{F}\right) = 0$$
$$\nabla\cdot\mathbf{u}_{\mathrm{br}} = 0 \qquad (1)$$

where ρ is density (kg/m^3), η gives the dynamic viscosity (kg/(m·s)) **u** equals the velocity vector (m/s), p is pressure (kg/(m·s)), ε is the porosity, and k (m^2) denotes the permeability. The equation can account for the influence of small gravity and compressibility effects in the force term, **F** (N/m^3), which in this example equals zero. Some argue that k in the Brinkman equations differs slightly from κ in Darcy's law (Ref. 2), but this example calls for the same permeability in both flow zones.

From the Brinkman side of the Darcy–Brinkman interface you constrain velocity because the boundary condition on the Darcy side fixes the pressure. The velocity constraint on the Brinkman side of the interface reflects that velocities are dependent variables in the Brinkman equations but not in Darcy's law. The boundary condition on velocities is

$$\mathbf{u}_{\mathrm{br}} = \mathbf{u}_{\mathrm{dl}}$$

To implement the condition, use the Darcy velocities, $\mathbf{u}_{\mathrm{dl}}$, that COMSOL Multiphysics automatically calculates.

The confining layer and well casing are impermeable to flow. Approximate this situation with the no-slip condition

$$\mathbf{u}_{\mathrm{br}} = \mathbf{0}$$

This equation eliminates all components of the velocity vector at the boundary.

Getting a unique solution to this problem requires defining the pressure at the well because the model prescribes flux conditions for all other boundaries. The constraint on pressure is the simple statement that

$$p_{\mathrm{br}} = p_{\mathrm{well}}$$

For the Brinkman problem, the boundary conditions are now

$$\begin{aligned}
\mathbf{u}_{\mathrm{br}} &= \mathbf{u}_{\mathrm{dl}} && \partial\Omega\ \text{Darcy-Brinkman interface}\\
\mathbf{u}_{\mathrm{br}} &= \mathbf{0} && \partial\Omega\ \text{Confining layers}\\
\mathbf{u}_{\mathrm{br}} &= \mathbf{0} && \partial\Omega\ \text{Well casing}\\
p_{\mathrm{br}} &= p_{\mathrm{well}} && \partial\Omega\ \text{Perforation}
\end{aligned}$$

Modeling in COMSOL Multiphysics

The two equations in this analysis are fundamentally compatible as both describe fluid flow, pressure distributions, and velocities. Even so, the dependent variable in Darcy's law is pressure alone, whereas pressure and directional velocities are the dependent variables in the Brinkman equations. The difference in the number of dependent variables amounts to a slight incompatibility in form, which you circumvent by imposing non-ideal weak constraints on the equation system. The weak constraints provide new integral equations in which the Lagrange multipliers μ_1 are dependent variables. The constraints add one Lagrange multiplier to Darcy's law and two to the Brinkman equations to make up for the difference in the number of degrees of freedom given by the two governing equation systems on the boundaries. Adding the new Lagrange multipliers is easy: go to the **Application Mode Properties** dialog boxes, find the **Weak constraints** list, and select **On**. To make the weak constraints non-ideal, select **Non-ideal** from the **Constraint type** list.

To find out more about weak constraints and weak formulation equations, see "Using Weak Constraints" on page 350 in the *COMSOL Multiphysics Modeling Guide*.

Model Data

The data for parameterizing the model are:

VARIABLE	UNITS	DESCRIPTION	EXPRESSION
g_r	m/s^2	Acceleration due to gravity	9.82
ρ_f	kg/m^3	Fluid density	900
η_f	Pa·s	Dynamic viscosity	0.002
ε		Porosity	0.4
κ	m^2	Permeability	10^{-10}
b	m	Thickness of layer	1
r_{res}	m	Reservoir radius	4
r_w	m	Well radius	0.1
W	m^3/s	Pumping rate	10^{-3}
p_{well}	Pa	Pressure at perforation	105

Results and Discussion

Figure 2 shows the solution to the Darcy-Brinkman problem where Darcy's law governs slow flow far from the well, but near it the Brinkman equations apply. The impact of the switch between flow laws occurs at $x = 1$ m. The streamlines show the fluid moving from the inlet at the right to the well on the left. The streamlines funnel because the flow is moving into a break or perforation in the well casing.

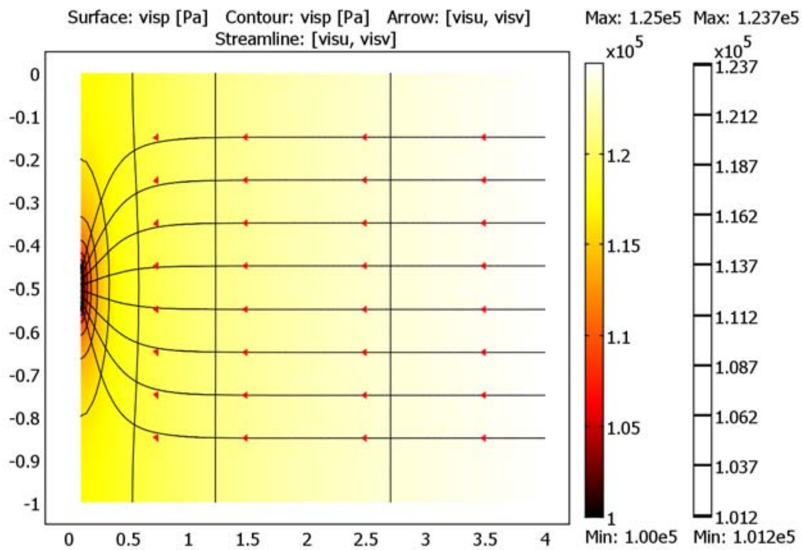

Figure 2: COMSOL Multiphysics solution for Darcy's law (1 m < r < 4 m) and the Brinkman equations (0.1 m < r < 1 m). The results shown are pressure (surface plot and contours) and velocities (streamlines). Note that the vertical axis is expanded.

Figure 3 and Figure 4, respectively, illustrate the pressure and velocity estimates from the perforation to a distance of 2 m beyond the Darcy-Brinkman interface. These estimates vary smoothly across the Brinkman-Darcy interface. Pressure increases with distance from the well, and it moves the fluid to the perforation. The velocities decrease with distance from the well until they reach an almost constant value in the

Darcy flow zone. The fact that the pressure estimates vary smoothly from Brinkman ($r < 1$ m) to Darcy ($r > 1$ m) flow indicates a stable solution.

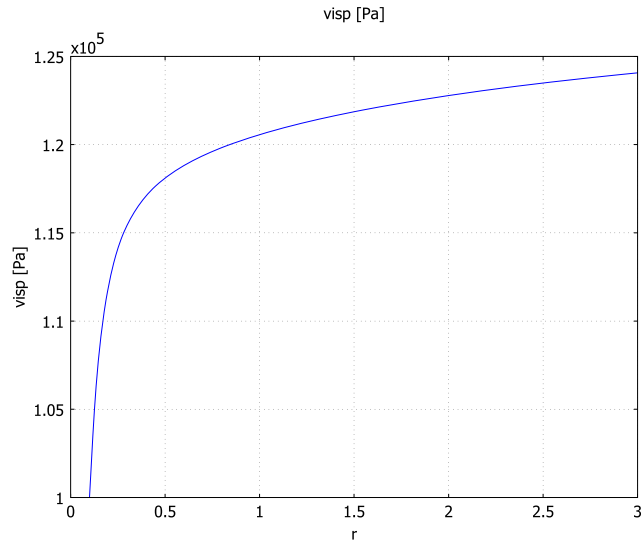

Figure 3: Pressure across the Darcy-Brinkman interface. Cross section along z = -0.5 m from r = 0.1 to 3.0 m.

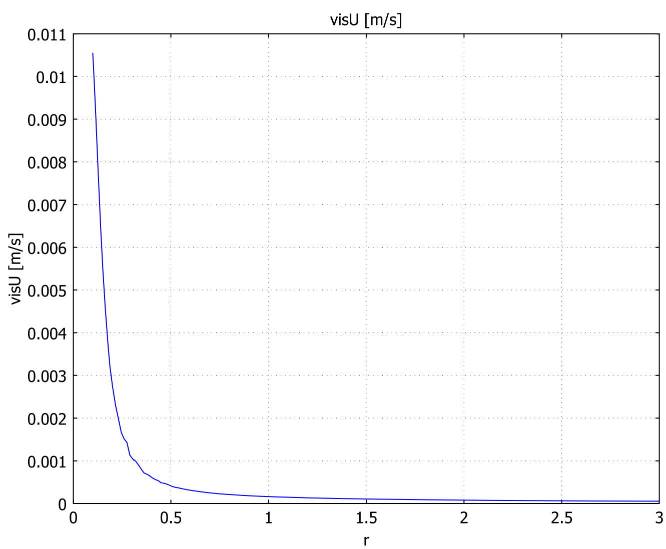

Figure 4: Velocity across the Darcy-Brinkman interface. Cross section along z = -0.5 m from r = 0.1 to 3.0 m.

This COMSOL Multiphysics example describes a straightforward protocol to couple two compatible flow laws with different dependent variables, the Darcy and Brinkman flow equations. Both Darcy's law and the Brinkman equations characterize flow in porous media. Because Darcy's law provides for no momentum transport by shear, it can overpredict flow rates in fast flow zones. Coupling to the Brinkman equations describes the added energy transformation.

The model is easy to modify and apply to a number of transitional flow scenarios including a river bottom, quickening flow near a well, and fluid moving in and around fractures. The next example adds the Navier-Stokes equations in the well to characterize the full transition between porous media and free surface flow.

References

1. M. Jamiolahmady, A. Danesh, D.H. Terhani, G.D. Henderson, and D.B. Duncan, "Flow around a rock perforation surrounded by damaged zone: Experiments vs. Theory," IASME/WSEAS Int'l Conf., Corfu, Greece, 2004.

2. U. Shavit, R. Rosenzweig, and S. Assouline, "Free flow at the interface of porous surfaces: A generalization of the Taylor Brush configuration," *Transport in Porous Media*, Kluwer Academic Publishers, 2003.