



For Domain 1:

$$\frac{\partial u_1}{\partial t} - \nabla^2 u_1 + u_1 \left(\frac{k_2^2}{u_1^4} (u_2 x^2 + u_2 y^2) + u_2^2 - 1 \right) = 0$$

$$-\nabla \cdot \left(\frac{k_2 \nabla u_2}{u_1^2} \right) + \frac{u_2}{k_2} = 0$$

For Domain 2:

$$\frac{\partial u_1}{\partial t} - \left(\frac{\epsilon_1^2}{\epsilon_2^2} \right) \nabla^2 u_1 + u_1 \left(\frac{\epsilon_1^2 k_1^2}{u_1^4} (u_2 x^2 + u_2 y^2) + u_2^2 - 1 \right) = 0$$

$$-\nabla \cdot \left(\frac{k_1 \nabla u_2}{u_1^2} \right) + \frac{u_2}{k_1} = 0$$

Where $k_1, k_2, \epsilon_1, \epsilon_2$ are constant. Boundary condition at 6 and 7

$$\frac{k_2}{u_1^2} \frac{\partial u_2}{\partial x} = \frac{k_1}{u_1^2} \frac{\partial u_2}{\partial x}$$

With

$$\frac{\partial u_2}{\partial y} = \frac{\partial u_2}{\partial y} = 0 \text{ and } u_2 = u_2 \text{ and}$$

$$\frac{\partial u_1}{\partial x} \Big|_{\text{domain1}} = \frac{\partial u_1}{\partial x} \Big|_{\text{domain2}} = 0$$