Self-Focusing

Introduction

For low intensities, the refractive index is independent of the intensity of the light in the material. However, when the intensity is large, so large that the electric field of the light field actually start to perturb the electron clouds around the nuclei, the refractive index start to depend on the intensity. Thus, for dielectrics, like glass, the refractive index increases with the intensity.

When a Gaussian beam propagates through a medium with an intensity-dependent refractive index, the index will be highest at the center of the beam. Thus, the induced refractive index profile will act as a lens or a waveguide that counteracts the spreading of the beam due to diffraction. The effect that the beam itself induces this positive, focusing, lens in the material is called self-focusing.

Self-focusing manifests itself both in terms of whole-beam focusing, where the beam's properties are changed by the induced index profile, and by small-scale focusing, where noise across the beam's cross-sectional intensity distribution is amplified and the beam can break up in to several self-focused filaments.

The nonlinear refractive index is written as

$$n = n_0 + \gamma I ,$$

where n_0 is the constant (linear) part of the refractive index, γ is the nonlinear refractive index coefficient and I is the intensity. The nonlinearity is due to the optical Kerr effect, which is a nonlinear distortion of the electron clouds around the nuclei in the material. For the standard optical glass BK-7, the nonlinear coefficient is 4×10^{-16} cm^2/W. Thus, for an intensity of 2.5 GW/cm^2, the induced refractive index change is 10^{-6}. This is a small change, but, as you will see, it will have a significant effect on the beam parameters.

Self-focusing is important from a laser engineering perspective, as the modification of the beam must be incorporated in the design. Furthermore, if the threshold for self-focusing is exceeded, the material will be damaged. Thus, it important to know the self-focusing threshold values for the materials used in the design. Self-focusing occurs in dielectrics, like optical glasses and laser rod materials, such as Nd:YAG.

A first estimate of the threshold power for self-focusing is obtain by assuming that the beam has a circular cross-section with a uniform intensity. Within this beam, the refractive index will be higher than outside the beam. Thus, the beam itself induces a waveguide structure. You can equal the acceptance angle of the waveguide with the diffraction angle from the circularly confined light. From this equality, you get the critical power to be

$$P_{cr} = (1.22\pi)^2 \frac{\lambda_0^2}{8\pi n_0 \gamma}. \tag{1}$$

Model Definition

The geometry for the model is simple - just a cylinder.

The incident beam is approximated by a Gaussian beam, polarized in the z-direction. The electric field is given by

$$\mathbf{E}(x, y, z) = E_0 \frac{w_0}{w(x)} \exp\left(-\frac{y^2 + z^2}{w^2(x)}\right) \exp\left(-jk\frac{y^2 + z^2}{2R(x)}\right) \exp(-j(kx - \eta(x)))\mathbf{z},$$

where E_0 is the electric field amplitude, w_0 is the spot radius at the waist, k is the wave number, defined by $k = 2\pi n_0/\lambda$, and $\mathbf{z}$ is the unit vector in the z-direction. The function $w(x)$ defines the spot size variation as a function of the distance from the beam waist,

$$w(x) = w_0 \sqrt{1 + \frac{x^2}{x_0^2}}, \tag{2}$$

where $x_0 = \pi n_0 w_0{}^2/\lambda$ is the Rayleigh range. The radius of curvature is defined by

$$R(x) = x\left(1 + \frac{x_0^2}{x^2}\right)$$

and the phase change close to the beam waist, the so called Gouy shift, is defined by

$$\eta(x) = \mathrm{atan}\left(\frac{x}{x_0}\right).$$

Results and Discussion

Figure 1 shows the Gaussian beam for a low input intensity. As expected, the beam is symmetric around the beam waist location.

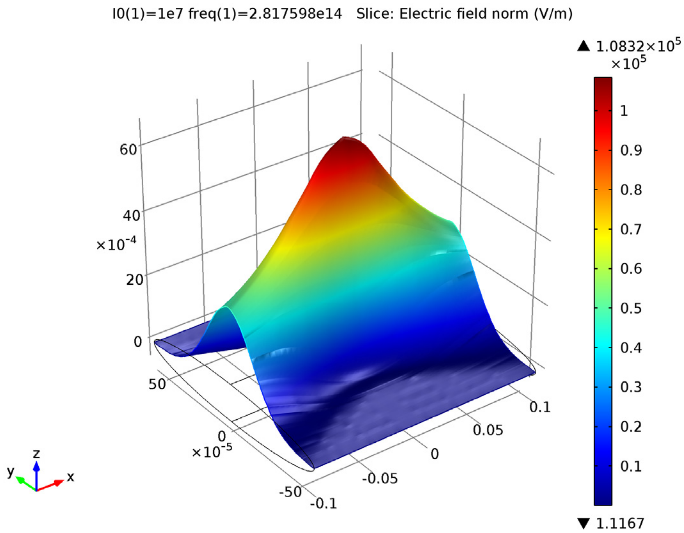


Figure 1: The Gaussian beam for a low peak intensity, $I_0 = 1$ *kW/cm*^{2}.

Figure 2 shows the beam for a high input intensity, I_0 = 6 GW/cm^2. The nominal induced refractive index, γI_0, is 2.4×10^{-6}. As noted from Figure 3 the actual induced refractive index is more than ten times the nominal value. This is of course an effect of the self-focusing of the beam.

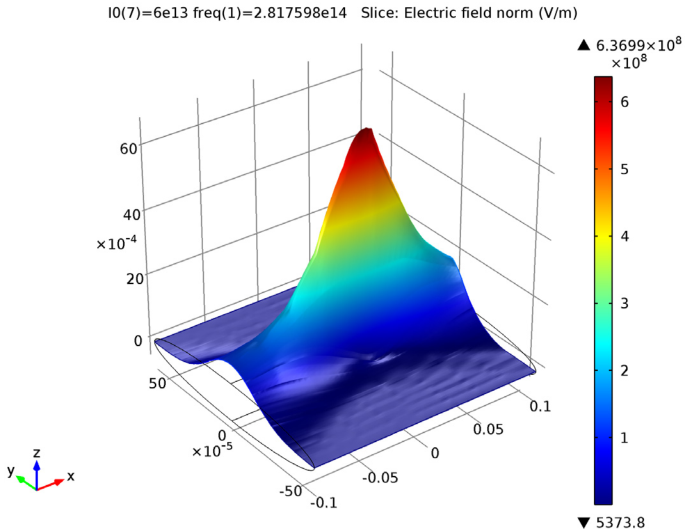


Figure 2: The Gaussian beam for a high peak intensity, $I_0 = 6\ GW/cm^2$.

To demonstrate the effect of self-focusing, Figure 4 shows the calculated spot radius at the boundary between the propagation domain and the PML domain. The spot radius is defined by

$$w = \sqrt{\frac{2\int_A I(y,z)(y^2+z^2)dydz}{\int_A I(y,z)dydz}}, \qquad (3)$$

where A is the integration area and $I(y,z)$ is the cross-sectional intensity distribution of the beam. For low peak intensities, the calculated spot radius, using Equation 3, give similar results as the Gaussian beam expression in Equation 2. However, with increasing intensities, the beam start to deviate from a Gaussian beam and the spot radius is reduced linearly with intensity.

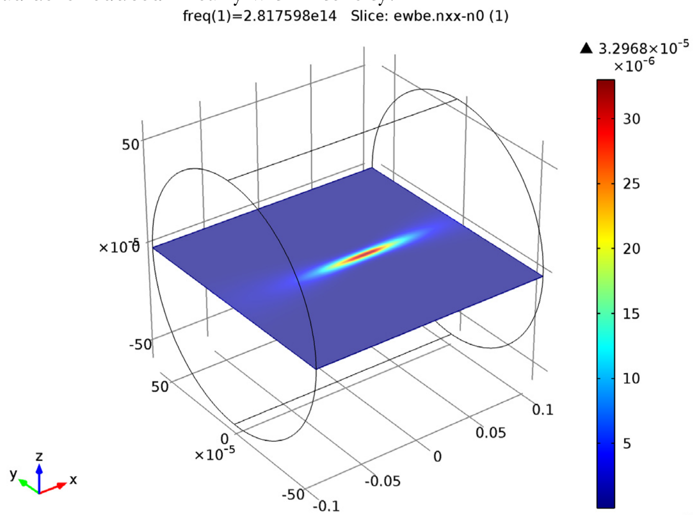


Figure 3: The induced refractive index change, γI, for a high peak intensity, $I_0 = 6$ *GW/cm*^{2}.

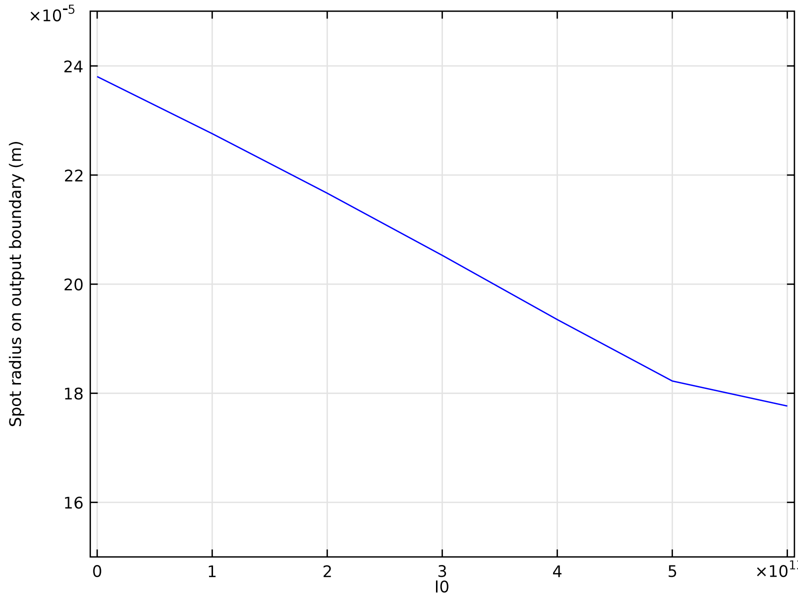


Figure 4: The spot radius at the end of the propagation domain versus the peak intensity.

The final intensity used in the parametric sweep is 6 GW/cm^2. This corresponds to a power that is 22% of the critical power, provided in Equation 1. As discussed in Ref. 1, it is expected that the critical power for a Gaussian beam is reduced by a factor of approximately $4/(1.22\pi)^2 = 0.27$. Thus, the power corresponding to the last peak intensity in the sweep is approximately a factor $0.22/0.27 = 0.81$ from the critical power for self-focusing for a Gaussian beam, where the induced refractive index profile completely balances the diffractive spreading of the beam.

To compute the field for even higher intensities, a much finer mesh would be needed, as the beam can break up into filaments. For a nonlinear problem like this, it is important to verify the results by, for instance, repeating the simulation with a finer mesh.

Reference

1. W. Koechner, *Solid-State Laser Engineering*, Springer, chap. 4.6, 2010.

Model Library path: `Wave_Optics_Module/Nonlinear_Optics/self_focusing`

Modeling Instructions

From the **File** menu, choose **New**.

NEW

1 In the **New** window, click the **Model Wizard** button.

MODEL WIZARD

1 In the **Model Wizard** window, click the **3D** button.

2 In the **Select physics** tree, select **Optics>Wave Optics>Electromagnetic Waves, Beam Envelopes (ewbe)**.

3 Click the **Add** button.

4 Click the **Study** button.

5 In the tree, select **Preset Studies>Frequency Domain**.

6 Click the **Done** button.

GLOBAL DEFINITIONS

Start by adding some global model parameters.

Parameters

1 On the **Home** toolbar, click **Parameters**.

2 In the **Parameters** settings window, locate the **Parameters** section.

3 In the table, enter the following settings:

Name	Expression	Value	Description
wl	1.064[um]	1.064E-6 m	Wavelength
f0	c_const/wl	2.818E14 1/s	Frequency
w0	100*wl	1.064E-4 m	Nominal spot radius
n0	1.52	1.520	Refractive index of BK-7 glass
x0	pi*n0*w0^2/wl	0.05081 m	Rayleigh range
k	2*pi*n0/wl	8.976E6 1/m	Propagation constant

Name	**Expression**	**Value**	**Description**
`I0`	`2.5[GW/cm^2]`	2.500E13 W/m²	Nominal peak intensity
`E0`	`sqrt(2*I0/ n0*sqrt(mu0_const/ epsilon0_const))`	1.113E8 V/m	Nominal peak electric field
`length`	`4*x0`	0.2032 m	Length of computation domain
`radius`	`2.5*w0*sqrt(1+(length/ (2*x0))^2)`	5.948E-4 m	Radius of computation domain
`gamma`	`4e-16[cm^2/W]`	4.000E-20 s³/kg	Nonlinear refractive index coefficient
`delta_n`	`gamma*I0`	1.000E-6	Nominal refractive index change
`P_cr`	`(1.22*pi)^2*wl^2/ (8*pi*n0*gamma)`	1.088E7 W	Critical power

Now add the functions that describe the incident Gaussian beam. Start with the function describing the spot radius versus distance from focus.

Analytic 1

1. On the **Home** toolbar, click **Functions** and choose **Global>Analytic**.
2. In the **Analytic** settings window, locate the **Function Name** section.
3. In the **Function name** edit field, type `w`.
4. Locate the **Definition** section. In the **Expression** edit field, type `w0*sqrt(1+(x/x0)^2)`.
5. Locate the **Units** section. In the **Arguments** edit field, type `m`.
6. In the **Function** edit field, type `m`.

Next, add the radius of curvature function.

Analytic 2

1. On the **Home** toolbar, click **Functions** and choose **Global>Analytic**.
2. In the **Analytic** settings window, locate the **Function Name** section.
3. In the **Function name** edit field, type `R`.
4. Locate the **Definition** section. In the **Expression** edit field, type `x*(1+(x0/x)^2)`.
5. Locate the **Units** section. In the **Arguments** edit field, type `m`.
6. In the **Function** edit field, type `m`.

Finally, add the function describing the phase shift around the focus region.

Analytic 3

1 On the **Home** toolbar, click **Functions** and choose **Global>Analytic**.

2 In the **Analytic** settings window, locate the **Function Name** section.

3 In the **Function name** edit field, type `eta`.

4 Locate the **Definition** section. In the **Expression** edit field, type `atan2(x,x0)`.

5 Locate the **Units** section. In the **Arguments** edit field, type `m`.

GEOMETRY 1

Cylinder 1

1 On the **Geometry** toolbar, click **Cylinder**.

2 In the **Cylinder** settings window, locate the **Size and Shape** section.

3 In the **Radius** edit field, type `radius`.

4 In the **Height** edit field, type `length`.

5 Locate the **Position** section. In the **x** edit field, type `-length/2`.

6 Locate the **Axis** section. From the **Axis type** list, choose **x-axis**.

7 On the **Home** toolbar, click **Build All Objects**.

GLOBAL DEFINITIONS

Since the geometry is so long and thin, modify the view setting to not preserve the aspect ratio.

DEFINITIONS

Camera

1 In the **Model Builder** window, expand the **Component 1>Definitions>View 1** node, then click **Camera**.

2 In the **Camera** settings window, locate the **Camera** section.

3 Clear the **Preserve aspect ratio** check box.

4 Click the **Apply** button.

5 Click the **Zoom Extents** button on the Graphics toolbar.

MATERIALS

Now add the BK-7 glass used in the model.

Material 1

1 In the **Model Builder** window, under **Component 1** right-click **Materials** and choose **New Material**.

2 In the **Material** settings window, locate the **Material Contents** section.

3 In the table, enter the following settings:

Property	Name	Value	Unit	Property group
Refractive index	n	`n0+gamma*ewbe.Poavx`	1	Refractive index

The variable `ewbe.Poavx` represents the intensity in the propagation direction.

4 Right-click **Component 1>Materials>Material 1** and choose **Rename**.

5 Go to the **Rename Material** dialog box and type `BK-7 glass` in the **New name** edit field.

6 Click **OK**.

DEFINITIONS

Setup a boundary integration operator, for calculation of the output power and the output spot radius.

Integration 1

1 On the **Definitions** toolbar, click **Component Couplings** and choose **Integration**.

2 In the **Integration** settings window, locate the **Operator Name** section.

3 In the **Operator name** edit field, type `intop_output_boundary`.

4 Locate the **Source Selection** section. From the **Geometric entity level** list, choose **Boundary**.

5 Select Boundary 6 only.

Add the expressions for the output power and the output spot radius.

Variables 1

1 In the **Definitions** toolbar, click **Local Variables**.

2 In the **Variables** settings window, locate the **Variables** section.

3 In the table, enter the following settings:

Name	Expression	Unit	Description
P	intop_output_boundary(ewbe.nPoav)	W	Output power
w_t	sqrt(2*intop_output_boundary(ewbe.nPoav*(y^2+z^2))/P)	m	Spot radius on output boundary

ELECTROMAGNETIC WAVES, BEAM ENVELOPES

Set the interface to use unidirectional propagation and define the wave vector component in the x-direction.

1 In the **Model Builder** window, under **Component 1** click **Electromagnetic Waves, Beam Envelopes**.

2 In the **Electromagnetic Waves, Beam Envelopes** settings window, locate the **Wave Vectors** section.

3 From the **Number of directions** list, choose **Unidirectional**.

4 Specify the $\mathbf{k}_1$ vector as

k	x
0	y
0	z

Use a matched boundary condition to launch an incident Gaussian beam polarized in the z-direction.

Matched Boundary Condition 1

1 On the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 1 only.

3 In the **Matched Boundary Condition** settings window, locate the **Matched Boundary Condition** section.

4 From the **Incident field** list, choose **Electric field**.

5 Specify the $\mathbf{E}_0$ vector as

0	x
0	y
E0*w0/w(x)*exp(-(y^2+z^2)/w(x)^2)*exp(-i*(k*x-eta(x)+k*(y^2+z^2)/(2*R(x))))	z

Matched Boundary Condition 2

1 On the **Physics** toolbar, click **Boundaries** and choose **Matched Boundary Condition**.

2 Select Boundary 6 only.

MESH 1

1 In the **Model Builder** window, under **Component 1** click **Mesh 1**.

2 In the **Mesh** settings window, locate the **Mesh Settings** section.

3 From the **Sequence type** list, choose **User-controlled mesh**.

Set the main **Size** settings to represent the discretization along the propagation direction. Since the beam is expected to behave as a slightly distorted Gaussian beam, it is sufficient to set the maximum mesh element size to half the Rayleigh range.

Size

1 In the **Model Builder** window, under **Component 1>Mesh 1** click **Size**.

2 In the **Size** settings window, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. In the **Maximum element size** edit field, type `x0/2`.

Add a free triangular mesh on the input surface. This mesh should resolve the beam's cross-sectional distribution.

Free Triangular 1

1 In the **Model Builder** window, right-click **Mesh 1** and choose **More Operations>Free Triangular**.

2 Select Boundary 1 only.

Size 1

1 Right-click **Component 1>Mesh 1>Free Triangular 1** and choose **Size**.

2 In the **Size** settings window, locate the **Element Size** section.

3 Click the **Custom** button.

4 Locate the **Element Size Parameters** section. Select the **Maximum element size** check box.

5 In the associated edit field, type `w0/2`.

6 Select the **Minimum element size** check box.

7 In the associated edit field, type `w0/4`.

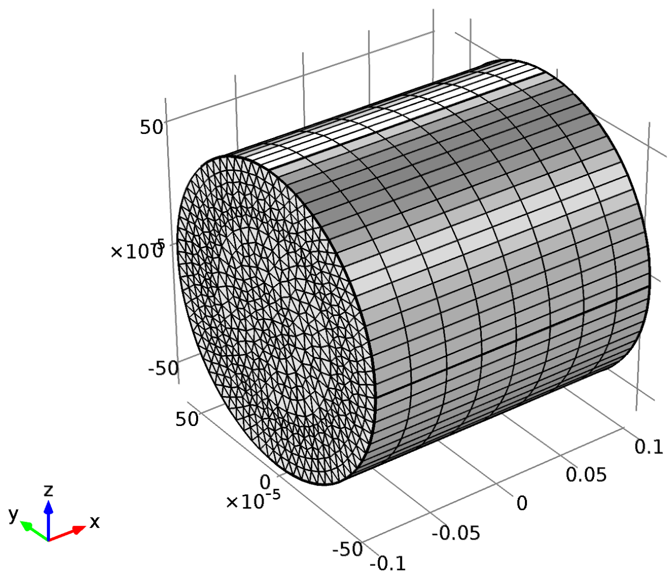
Now remove the default **Free Tetrahedral 1** node and replace it with a **Swept** mesh node.

Free Tetrahedral 1

1 In the **Model Builder** window, under **Component 1>Mesh 1** right-click **Free Tetrahedral 1** and choose **Delete**. Click **Yes** to confirm.

2 Right-click **Mesh 1** and choose **Swept**.

3 Right-click **Mesh 1** and choose **Build All**.



STUDY 1

Don't generate the default plots.

1 In the **Model Builder** window, click **Study 1**.

2 In the **Study** settings window, locate the **Study Settings** section.

3 Clear the **Generate default plots** check box.

Step 1: Frequency Domain

1 In the **Model Builder** window, under **Study 1** click **Step 1: Frequency Domain**.

2 In the **Frequency Domain** settings window, locate the **Study Settings** section.

3 In the **Frequencies** edit field, type `f0`.

Setup a parametric sweep of the nominal peak intensity, from 1 kW/cm^2 (corresponding to linear propagation) to 6 GW/cm^2 that will show a significant self-focusing effect.

Parametric Sweep

1 On the **Study** toolbar, click **Parametric Sweep**.
2 In the **Parametric Sweep** settings window, locate the **Study Settings** section.
3 Click the **Add** button.
4 In the table, enter the following settings:

Parameter names	Parameter value list
I0	1e7, 1e13, 2e13, 3e13, 4e13, 5e13, 6e13

5 Click to expand the **Study extensions** section. Locate the **Study Extensions** section. From the **Use parametric solver** list, choose **Off**, to turn off the parametric solver that otherwise would perform calculations also for intermediate intensities.

Since this is a nonlinear problem, it is better to split the complex electric field variable into its real and imaginary parts. This will produce a more accurate Jacobian for the problem, leading to a faster convergence.

Solver 1

1 On the **Study** toolbar, click **Show Default Solver**.
2 In the **Model Builder** window, expand the **Solver 1** node, then click **Compile Equations: Frequency Domain**.
3 In the **Compile Equations** settings window, locate the **Study and Step** section.
4 Select the **Split complex variables in real and imaginary parts** check box.
5 On the **Study** toolbar, click **Compute**.

RESULTS

3D Plot Group 1

1 On the **Results** toolbar, click **3D Plot Group**.
2 In the **3D Plot Group** settings window, locate the **Data** section.
3 From the **Data set** list, choose **Solution 2**.
4 On the **3D Plot Group 1** toolbar, click **Slice**.
5 In the **Slice** settings window, locate the **Plane Data** section.
6 From the **Plane** list, choose **xy-planes**.

7 In the **Planes** edit field, type `1`.
8 Right-click **Results>3D Plot Group 1>Slice 1** and choose **Deformation**.
9 In the **Deformation** settings window, locate the **Expression** section.
10 In the **z component** edit field, type `ewbe.normE`.
11 Click the **Go to Default 3D View** button on the Graphics toolbar.

Take a look at the field distributions for the different intensities.

12 In the **Model Builder** window, click **3D Plot Group 1**.
13 In the **3D Plot Group** settings window, locate the **Data** section.
14 From the **Parameter value (I0)** list, choose the desired intensity.
15 On the **3D plot group** toolbar, click **Plot**.

Compare the graph at the intensity **6e13** with that in Figure 2.

To really see how the beam changes with intensity, you can also visualize this by running the 3D plots in the Player. The first step would be to normalize the field solution with the nominal peak electric field.

16 In the **Model Builder** window, under **Results>3D Plot Group 1** click **Slice 1**.
17 In the **Slice** settings window, locate the **Expression** section.
18 In the **Expression** edit field, type `ewbe.normE/E0`.
19 In the **Model Builder** window, under **Results>3D Plot Group 1>Slice 1** click **Deformation 1**.
20 In the **Deformation** settings window, locate the **Expression** section.
21 In the **z component** edit field, type `ewbe.normE/E0`.

Export

1 On the **Results** toolbar, click **Player**.
2 In the **Player** settings window, locate the **Animation Editing** section.
3 From the **Loop over** list, choose **I0**.
4 Locate the **Playing** section. In the **Display each frame for** edit field, type `1`.
5 Right-click **Results>Export>Player 1** and choose **Play**. If you check the **Repeat** box in the **Playing** settings, you can have the Player repeat the sequence.

Now create a new plot of the refractive index change induced by the beam.

3D Plot Group 2

1 On the **Results** toolbar, click **3D Plot Group**.

2 On the **Results** toolbar, click **Slice**.
3 In the **Slice** settings window, locate the **Plane Data** section.
4 From the **Plane** list, choose **xy-planes**.
5 In the **Planes** edit field, type `1`.
6 Click **Replace Expression** in the upper-right corner of the **Expression** section. From the menu, choose **Electromagnetic Waves, Beam Envelopes>Material properties>Refractive index>Refractive index, xx component (ewbe.nxx)**.
7 Locate the **Expression** section. In the **Expression** edit field, type `ewbe.nxx-n0`.
8 On the **3D Plot Group 2** toolbar, click **Plot**.
9 Click the **Go to Default 3D View** button on the Graphics toolbar. Compare your graph with Figure 3.

Now, visualize how the spot radius decreases with intensity, using a table and a table plot.

Derived Values

1 On the **Results** toolbar, click **Global Evaluation**.
2 In the **Global Evaluation** settings window, locate the **Expression** section.
3 In the **Expression** edit field, type `w_t`.
4 Locate the **Data** section. From the **Data set** list, choose **Solution 2**.
5 Click the **Evaluate** button.

TABLE

In the **Table** window, click **Table Graph**.

RESULTS

1D Plot Group 3

1 In the **Model Builder** window, under **Results>1D Plot Group 3** click **Table Graph 1**.
2 In the **Table Graph** settings window, locate the **Data** section.
3 From the **x-axis data** list, choose **I0**.
4 From the **Plot columns** list, choose **Manual**.
5 In the **Columns** list, select **Spot radius on output boundary (m)**.
6 In the **Model Builder** window, click **1D Plot Group 3**.
7 In the **1D Plot Group** settings window, locate the **Axis** section.
8 Select the **Manual axis limits** check box.

9 In the **y minimum** edit field, type `1.5e-4`.

10 In the **y maximum** edit field, type `2.5e-4`.

11 Locate the **Grid** section. Select the **Manual spacing** check box.

12 In the **x spacing** edit field, type `1e13`.

13 In the **y spacing** edit field, type `0.2e-4`.

14 On the **1D Plot Group 3** toolbar, click **Plot**. Your result should look similar to that in Figure 4.

Now, compare the result with the output spot radius of the ideal Gaussian beam.

Derived Values

1 On the **Results** toolbar, click **Global Evaluation**.

2 In the **Global Evaluation** settings window, locate the **Expression** section.

3 In the **Expression** edit field, type `w(length/2)`.

4 Click the **Evaluate** button. You should find that the spot radius for the low intensity cases are close to that of the nominal beam.

RESULTS

Derived Values

Finally, compare the total power in the beam with the critical power for self-focusing, as defined in Equation 1.

1 On the **Results** toolbar, click **Global Evaluation**.

2 In the **Global Evaluation** settings window, locate the **Expression** section.

3 In the **Expression** edit field, type `P/P_cr`.

4 Locate the **Data** section. From the **Data set** list, choose **Solution 2**.

5 Right-click **Results>Derived Values>Global Evaluation 2** and choose **Evaluate>New Table**. You should find that the power ratio is approximately 22%.