SOLUTION TO COMPUTER LAB EXAM - 2.1

30-05-2013   **25 marks**

PROBLEM:

1. A rod of radius κR ($0 < \kappa < 1$) moves upward with a constant velocity v_0, through a cylindrical container of inner radius R containing a Newtonian liquid of density ρ and viscosity μ, as shown in Fig. 1 below. The liquid circulates in the cylinder, moving upward along the moving central rod and moving downward along the fixed container wall. Flows similar to this occur in the seals of some reciprocating machinery–for example, in the annular space between piston rings.

   Use COMSOL Multiphysics to find the steady-state velocity distribution in the annular region as a function of r and z. The cylindrical container has a radius $R = 2$ cm, and a height $L = 10$ cm. The rod radius is $0.25R$ (i.e., $\kappa = 0.25$). The 2D-axisymmetric geometry that can be used to solve the problem in COMSOL. The fluid properties, and velocity of the rod are,

   $$\rho = 800 \, \text{kg/m}^3; \; \mu = 0.16 \, \text{Pa} \cdot \text{s}; \; v_0 = 3 \, \text{cm/s}$$

   Introduce the dimensionless variables:

   $$\xi = \frac{r}{R}; \quad \zeta = \frac{z}{L}; \quad \phi = \frac{v_z}{v_0}$$

   (a) Display a surface plot of velocity in the tube, with a colour bar indicating the velocity. On this plot, also display streamlines for the velocity field. In the streamline settings window, choose *uniform density* for "streamline positioning" with a *separating distance* equal to 0.02. You may have to choose an appropriate colour to make the streamlines visible. Export the streamline plot as an image file to be pasted in your Word document. Describe the main features of the velocity field, such as the existence of recirculation zones.

   (b) Plot the non-dimensional velocity ϕ as a function of non-dimensional radial location ξ, at the following values of ζ: $\{0.01, 0.05, 0.1, 0.15 \, \text{and} \, 0.2\}$. Label your plot axes, and identify the different curves in the legend to the plot.

   (c) The following analytical solution for the non-dimensional velocity profile as a function of ξ has been obtained by neglecting end effects:

   $$\phi = \frac{(1 - \xi^2)\left(1 + \dfrac{2\kappa^2}{1 - \kappa^2} \ln \kappa\right) + (1 - \kappa^2) \ln \xi}{(1 - \kappa^2) + (1 + \kappa^2) \ln \kappa}$$

   Plot the non-dimensional analytical velocity on the same plot as the velocity profiles obtained in part (b) above. Label your curve appropriately. Export the
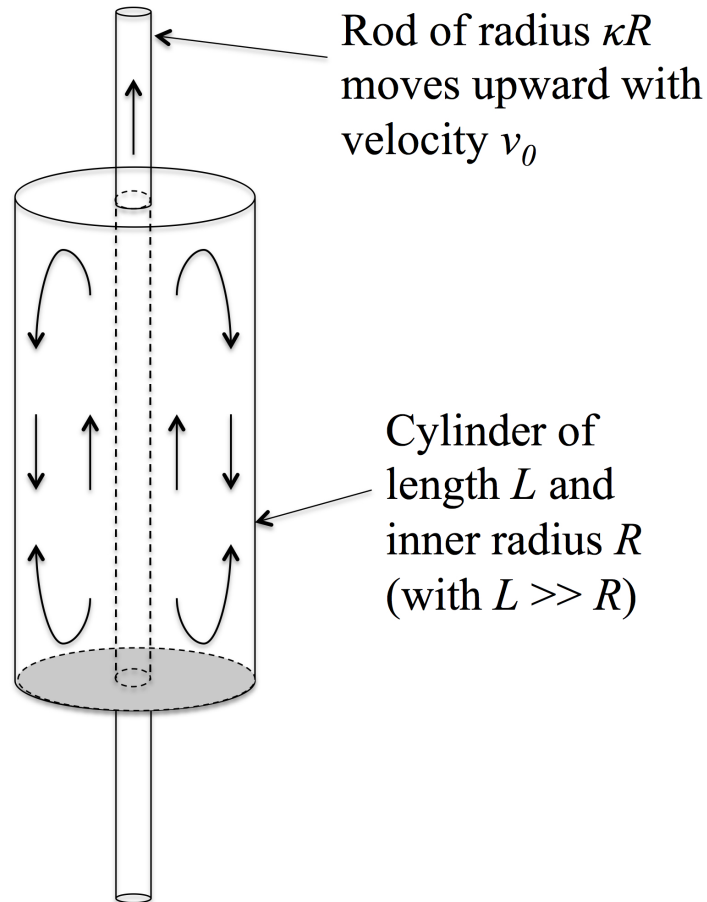
Figure 1: Circulating flow produced by an axially moving rod in a closed annular region.

image with the numerical and analytical velocity profiles for pasting in your Word document. Does the numerical velocity profile agree with the analytical profile at any of the values of ζ considered in part (b) above? What does this signify?

(d) Plot the non-dimensional velocity ϕ as a function of non-dimensional axial position ζ at $\xi = 0.1$. Label your plot axes, and identify the curve in the legend to the plot. Export the plot as an image file to be pasted in your Word document. By comparing this plot with the streamlines plot obtained earlier, point out the features of the velocity profile.

(e) With the help of a "coupling operator", evaluate the mean velocity across the cylinder cross-section at $\zeta = 0.5$. What does this imply for the net mass flow through the plane $\zeta = 0.5$?

(f) Generate an "Complete" report of your COMSOL solution as a html file, and upload the Word document and the COMSOL report together as a zipped file.

SOLUTION:

Step 1: Problem formulation on COMSOL

1. The problem clearly requires one to choose a 2D axisymmetric, Fluid Flow, Single-Phase Flow, Laminar Flow, Stationary package in COMSOL.

2. The parameters

$$\rho = 800 \, \text{kg/m}^3; \; \mu = 0.16 \, \text{Pa} \cdot \text{s}; \; v_0 = 3 \, \text{cm/s}; \; R = 0.02 \, \text{m}; \; L = 0.1 \, \text{m}; \; \kappa = 0.25$$

   can be entered as Parameters under Global Definitions.

3. Since we are asked to display results in terms of non-dimensional variables, we can enter the *numerically evaluated* non-dimensional variables,

$$\xi = \frac{r}{R}; \quad \zeta = \frac{z}{L}; \quad \phi = \frac{v_z}{v_0}$$

   and the *analytical expression* for the non-dimensional velocity,

$$\phi_{\text{anal}} = \frac{(1 - \xi^2)\left(1 + \dfrac{2\kappa^2}{1 - \kappa^2} \ln \kappa\right) + (1 - \kappa^2) \ln \xi}{(1 - \kappa^2) + (1 + \kappa^2) \ln \kappa}$$

   as Variables under Definitions. All the quantities in these equations have already been defined under Parameters.

4. The Geometry can now be constructed by creating two rectangles using the given dimensions and locations, followed by performing a Difference of the two rectangles as a Boolean Operation. Rectangle 1 has "Width = R' and "Height = L", located with its Corner at "r=0" and "z=0". Rectangle 2 has "Width = κR" and "Height = L", located with its Corner at "r=0", and "z=0".

5. A Material must then be added, and after choosing Domain 1, specify (under Material Contents) the Density and Viscosity as the defined parameters ρ and μ, respectively.

6. In the Laminar Flow option, select option All Domains and Incompressible flow.

7. Check that in the Fluid properties option, the "Density" and "Viscosity" are listed as "From material".

8. Choose "No slip" boundary conditions on all walls except the boundary corresponding to the moving rod.

9. At the inner cylinder wall specify the velocity v_0 as the z component of velocity.

10. Use a Physics-controlled, normal mesh.

11. The problem is now solvable on COMSOL.

Surface: Velocity magnitude (m/s) Arrow Surface: Velocity field
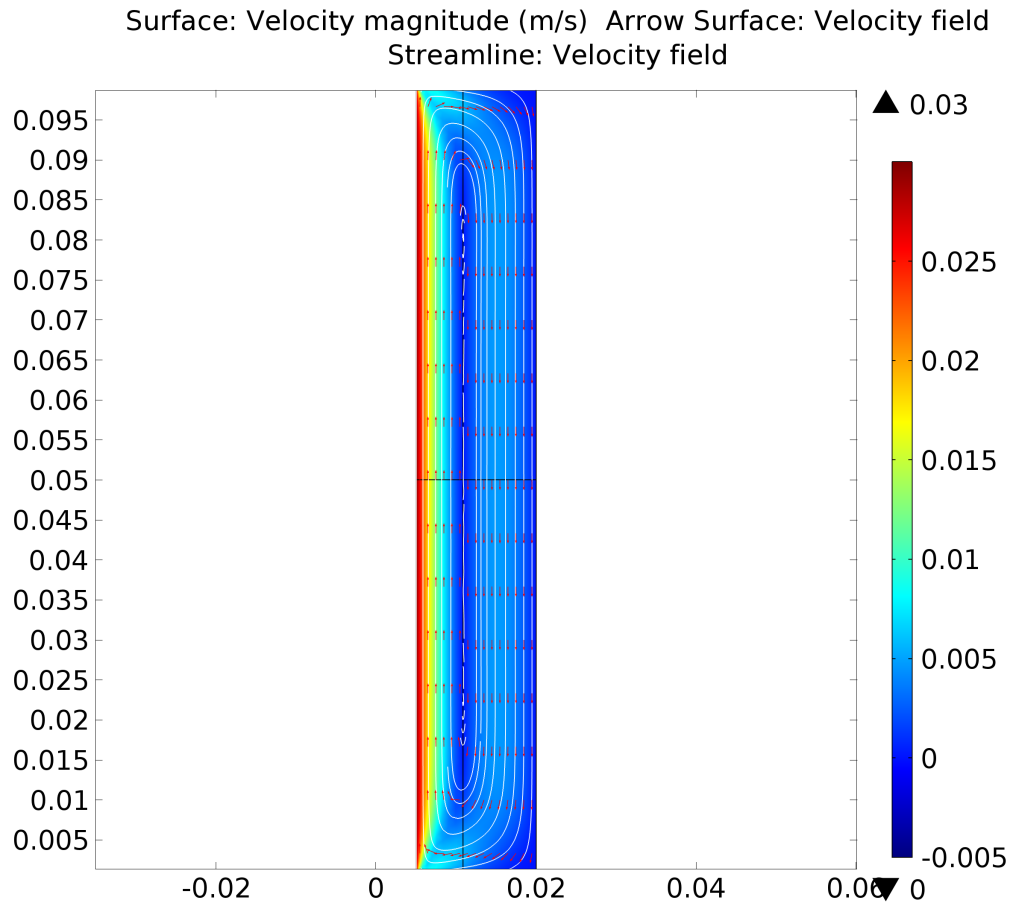**Streamline: Velocity field**

Figure 2: Streamlines

Step 2: Post processing results

1. A Surface plot of the velocity field, with a colour bar is immediately generated by COMSOL when the problem is solved. To obtain a streamline plot, right-click on Velocity and add Streamline. Choose the settings given in the problem to display the figure displayed as Fig. 2.

   It is clear that there is a large *recirculation* zone that occupies the entire cylindrical container, with fluid moving upward near in the middle of the cylinder dragged along by the rod, and flowing downwards close to the sides of the outer walls of the container.

2. Using the Cutline2D feature of COMSOL, non-dimensional velocity profiles at the desired values of $\zeta = \{0.01, 0.05, 0.1, 0.15 \text{ and } 0.2\}$ can be obtained. In the 1D Plot Group, Line Graph settings, choose the appropriate Cut Line under "data set" and define the y-axis as ϕ and the x-axis as ξ.

3. By adding a new Line Graph to the same 1D Plot Group, and defining the y-axis as ϕ_{anal} and the x-axis as ξ, the analytical velocity profile can be plotted on the same graph as the numerical profiles.
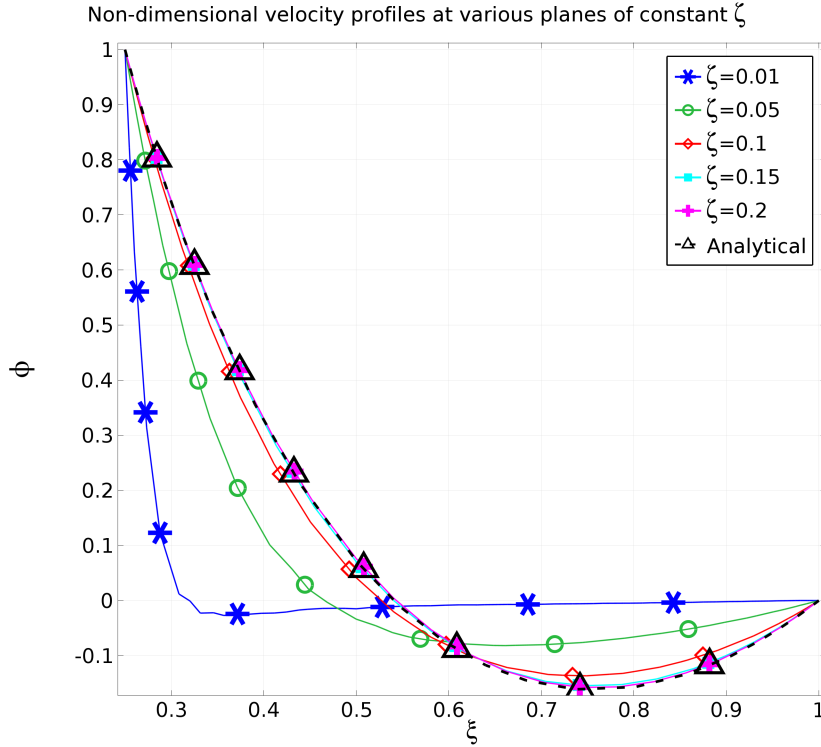
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Figure 3: Velocity profiles at various values of ζ as a function of ξ.

The results are displayed in the figure below.

It is clear that the analytical solution agrees with the numerical solution for values of $0.15 \leq \zeta \leq 0.85$ (because the top and bottom of the cylinder are symmetric). Since the analytical solution for the non-dimensional velocity profile has been obtained by neglecting end effects, this is not surprising. The numerical velocity profiles for values of ζ outside this range reflect the effects of the caps at the two ends of the cylinder where "no-slip" boundary conditions are imposed.

4. **This part of the problem was stated incorrectly**. "Plot the non-dimensional velocity ϕ as a function of non-dimensional axial position ζ at $\xi = 0.1$" should have been "Plot the non-dimensional velocity ϕ as a function of non-dimensional axial position ζ at $\xi = 0.5$" (which corresponds to $r = 0.01$). Any plot that you have plotted that represents the velocity profile as a function of ζ, at any fixed value of ξ, will be given full marks for this section.

   By "right-clicking" on Geometry and selecting Polygon, a vertical line can be drawn by selecting the range "r=0.01 0.01" and "z=0 0.01". The desired plot can be generated by using 1D Plot Group, Line Graph settings, choosing the polygon boundaries under "selection" and defining the y-axis as ϕ and the x-axis as ζ.

   Comparison of the figure below with the streamlines in Fig. 2 (and the profiles in Fig. 3) shows that as ξ increases from 0 (at the rod), at the radial location $\xi \approx 0.5$ (or $r = 0.01$), the fluid changes from moving upwards to downwards. We would therefore expect the velocity of the fluid to be small. This is seen in the figure

displayed below, with a further indication of the distance from the ends where a change in velocity direction occurs.
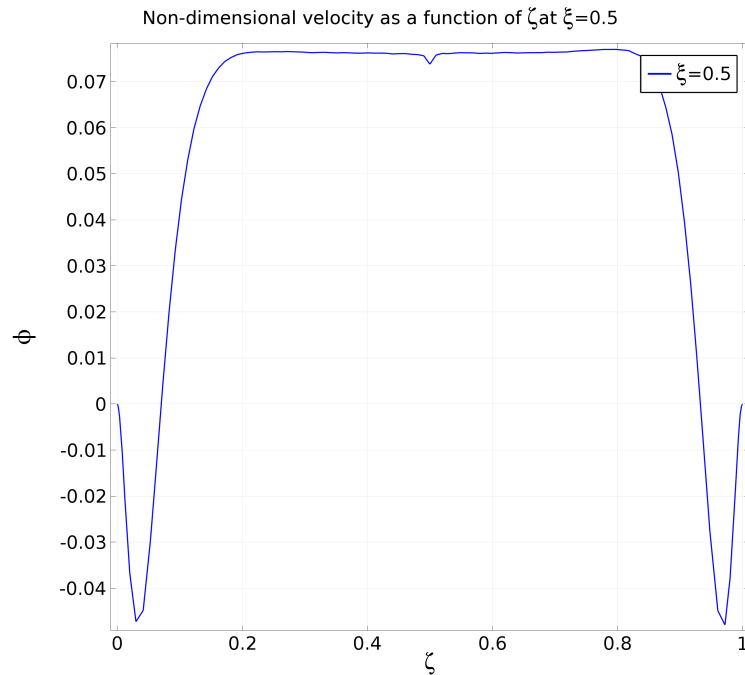

Figure 4: Velocity profile along the cylinder length at $\xi = 0.5$.

5. In order to evaluate the mean value of a property across any boundary, the following steps need to be followed:

   (a) By "right-clicking" on **Geometry** and selecting **Polygon**, a horizontal line can be drawn by selecting the range "r=0.005 0.02" and "z=0.05 0.05".

   (b) By "right-clicking" on **Definitions**, selecting **Model Couplings**, and **Average**, the average operator "aveop1" can be constructed. The boundary across which the average is to be carried out can be selected by picking the appropriate boundary in **Source Boundaries**.

   (c) Define the variable v_{mean} as a **Variable** under **Definitions**, and enter "aveop1(u)" under "Expression".

   (d) Finally under **Derived values**, pick **Global evaluation**, select v_{mean} under "Expression", and compute the solution again.

   This should lead to the following value of v_{mean}

   ```
   v_mean (m/s)
   1.30682e-7
   ```

   This clearly indicates that the mean velocity is zero, and that the net mass flow across the plane $\zeta = 0.5$ is zero. We expect this across any constant ζ plane since the fluid is incompressible and circulating in a confined geometry.