COMSOL MULTIPHYSICS®

Modeling Hysteresis Effects

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This document describes an approach to model hysteresis effects with COMSOL Multiphysics and the AC/DC Module. It is written for version 3.4 and later.

Introduction

Hysteresis effects originate from the alignment of how the electrons spin around the nucleus in a *magnetic domain*. For each domain there is a reversible and irreversible rotation, which corresponds to a reversible and irreversible magnetization (Ref. 1). In a magnetic material without hysteresis losses, there is only a reversible part of the magnetization, usually modeled with a relative permeability. A full hysteresis model includes a nonlinear relationship between the change in the magnetic field (**H**), the change in magnetization (**M**), and the change in the magnetic flux density (**B**). It is only necessary to define a relationship between two of these, because the third one can always be calculated with the formula

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

where μ_0 is the permeability for vacuum. One frequently cited method is the Jiles-Atherton method (Ref. 2), which this document base its modeling on. This model is also described in Ref. 1.

The full solution of a transient hysteresis model can be time consuming even for a 2D model, so it is often desirable to convert the hysteresis modeling to a reduced model. One such example is hysteresis losses in time-harmonic simulations. The hysteresis losses can in such cases be modeled using a complex valued permeability that represents the average energy loss over one cycle. As a final result, the time-harmonic loss parameter is extracted from the transient model described below.

Model Definition

The basic setup of the model is based on the inductor model defined in the model "Inductor in Amplifier Circuit" on page 191 of the *AC/DC Module Model Library.* The material model of the iron core for that model is replaced with a hysteresis model based on the Jiles-Atherton method. The inductor model uses the Azimuthal induction currents application mode of the AC/DC Module, solving for the magnetic potential **A**.

$$\sigma\frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\mu_0^{-1}\nabla \times \mathbf{A} - \mathbf{M}) = \mathbf{J}_e$$

where σ is the electric conductivity.

Because the inductor has a large number of turns it is not efficient to model each turn as a separate wire. Instead the entire coil is treated as a block with a constant external current density corresponding to the current in each wire. The conductivity in this block is zero to avoid eddy currents, which is motivated with the fact that no currents can flow between the individual wires. The eddy currents within each wire is neglected.

THE JILES-ATHERTON METHOD

More details about this method can be found in references 1, 2, and 3. The basic equations are

$$\mathbf{M} = \mathbf{M}_{\mathrm{irr}} + \mathbf{M}_{\mathrm{rev}}$$
$$\mathbf{M}_{\mathrm{rev}} = c(\mathbf{M}_{\mathrm{an}} - \mathbf{M}_{\mathrm{irr}})$$
$$\mathbf{M}_{\mathrm{an}} = M_s\left(\coth\frac{|\mathbf{H}_e|}{a} - \frac{a}{|\mathbf{H}_e|}\right)\frac{\mathbf{H}_e}{|\mathbf{H}_e|}$$
$$\frac{\partial \mathbf{M}_{\mathrm{irr}}}{\partial t} = \mathrm{g}\left(\frac{\mathbf{M}_{rev}}{ck}\cdot\frac{\partial \mathbf{H}_e}{\partial t}\right)\frac{\mathbf{M}_{rev}}{|\mathbf{M}_{rev}|} \quad ;\mathrm{g}(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$
$$\mathbf{H}_e = \mathbf{H} + \alpha\mathbf{M}$$

where a, c, α, M_s, and k are model parameters for the Jiles-Atherton (JA) method. A little algebra gives the following two important relations

$$\mathbf{M}_{\mathrm{rev}} = \frac{c}{1-c}(\mathbf{M}_{\mathrm{an}} - \mathbf{M})$$
$$\frac{\partial \mathbf{M}}{\partial t} = (1-c)\frac{\partial \mathbf{M}_{\mathrm{irr}}}{\partial t} + c\frac{\partial \mathbf{M}_{\mathrm{an}}}{\partial t}$$

where the second one is solved as an extra PDE with the magnetic field, $\mathbf{H}$, as solution variable (vector valued). The solution variable can be used directly in the constitutive relation **H=f(B)** for the Azimuthal application mode. It is equivalent to instead solve for the magnetization, $\mathbf{M}$, as solution variable and use the constitutive relation $\mathbf{B=\mu_0 H+\mu_0 M}$. The parameter values used in the model are specified in the table below.

Ref. 1 outlines a procedure to extract these parameters from measurements of magnetic materials.

PARAMETER	VALUE	UNIT
a	560	A/m
c	0.1	
α	0.0007	
M_s	1.6e6	A/m
k	1200	A/m

The function g(x) is implemented using the `flsmhs` function to avoid abrupt changes, using a transition parameter calculated with the following formula

$$\delta = \frac{cf^2}{k(1-c)}$$

where f is the frequency. Another numerical trick is also necessary when calculating the $\mathbf{M}_{\mathrm{an}}$ vector, because the given formula has singularities when $|\mathbf{H}_e|$ approaches zero. A standard Taylor expansion motivates the following approximation

$$\mathbf{M}_{\mathrm{an}} = \begin{cases} M_s\dfrac{\mathbf{H}_e}{3a} & ;|\mathbf{H}_e| < 0{,}1 \\ M_s\left(\coth\dfrac{|\mathbf{H}_e|}{a} - \dfrac{a}{|\mathbf{H}_e|}\right)\dfrac{\mathbf{H}_e}{|\mathbf{H}_e|} & ;|\mathbf{H}_e| \geq 0{,}1 \end{cases}$$

CONVERSION TO TIME-HARMONIC SIMULATION

In the equations below the magnitude of a vector is denoted with a dot product to avoid confusion with the complex absolute value.

A linear material model for losses in magnetic materials can be specified as follows

$$\mu_r = \mu_r' + j\mu_r''$$

where μ_r' and μ_r'' are the real part and imaginary part of the relative permeability. The power density in such a material can be calculated with

$$p_{\mathrm{loss}} = \frac{1}{2}\mathrm{Re}\{-j\omega\mathbf{B}\cdot\mathbf{H}^*\} = \frac{1}{2}\omega\mu_0\mu_r''(\mathbf{H}\cdot\mathbf{H}^*) = \omega\mu_0\mu_r''(\mathbf{H}_{\mathrm{rms}}\cdot\mathbf{H}_{\mathrm{rms}})$$

where $\mathbf{H}_{\mathrm{rms}}$ is the root-mean-square (rms) value of $\mathbf{H}$ over one cycle

$$\mathbf{H}_{\mathrm{rms}} = \sqrt{\frac{1}{t}\int_0^t \begin{bmatrix} H_x^2 \\ H_y^2 \\ H_z^2 \end{bmatrix} dt}$$

In the transient hysteresis model, you can calculate the power density with an integration over time

$$p_{\mathrm{loss}} = \frac{1}{t}\int_0^t \left(\mathbf{H}\cdot\frac{\partial\mathbf{B}}{\partial t}\right) dt$$

which will converge to an average power loss. Using the two equations for p_{loss} the time-harmonic parameters for the hysteresis material can be extracted from

$$\mathbf{B}_{\mathrm{rms}}\cdot\mathbf{B}_{\mathrm{rms}} = \mu_0^2|\mu_r|^2(\mathbf{H}_{\mathrm{rms}}\cdot\mathbf{H}_{\mathrm{rms}})$$

$$\mu_r'' = \frac{1}{2\pi f\mu_0}\frac{\frac{1}{V_{\mathrm{dom}}}\int_{\mathrm{dom}} p_{\mathrm{loss}} dV}{\frac{1}{V_{\mathrm{dom}}}\int_{\mathrm{dom}} (\mathbf{H}_{\mathrm{rms}}\cdot\mathbf{H}_{\mathrm{rms}}) dV}$$

$$\mu_r' = \mathrm{Re}\left\{\sqrt{\frac{1}{\mu_0^2}\frac{\int_{\mathrm{dom}} (\mathbf{B}_{\mathrm{rms}}\cdot\mathbf{B}_{\mathrm{rms}}) dV}{\int_{\mathrm{dom}} (\mathbf{H}_{\mathrm{rms}}\cdot\mathbf{H}_{\mathrm{rms}}) dV} - (\mu_r'')^2}\right\}$$

where dom is the domain representing the magnetic material. These equations give uniform parameters for the magnetic domain, but it is also possible to extract distributed parameters if the volume integrals are dropped. Such a detailed extraction only makes sense when the transient model is very similar to the time-harmonic model in terms of geometry and physics settings.

In the model, a global equation calculates the time integration of total magnetic energy over the magnetic material domain. All the rms values is calculated using the `quad` and `at`-operators. The `quad` operator performs a numerical integration, and the `at` operator evaluates an expression at a certain time. The rms integration must be done at the postprocessing stage, because the `at` operator cannot evaluate previous time steps during solving. These new variables are necessary to calculate the average total power loss and the time-harmonic parameters for the relative permeability. All the domain integrations are defined using integration coupling variables for subdomains.

Results and Discussion

The current in the coil is sinusoidal with a frequency of 50 Hz, and an amplitude of 5 A. In order to get a good average, the simulation runs for 80 ms, which is four full cycles. A plot of the z-component of the **B**-field versus the z-component of the **H**-field, shows a very pronounced hysteresis effect. The figure below shows such a plot where the fields are evaluated at the origin.

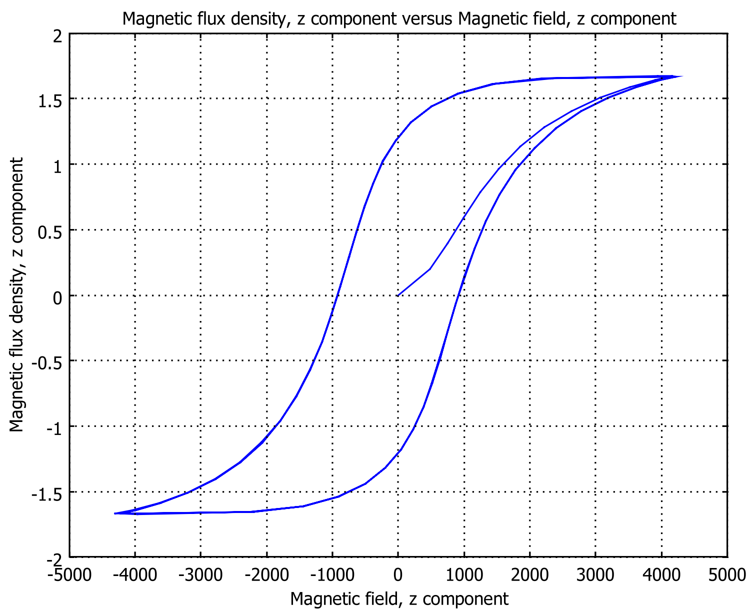

Figure 1: The hysteresis loop evaluated at the origin.

To get a value of the average power loss, the total magnetic energy inside the magnetic material is recorded. The average over time gives the power loss, which gets a more accurate result for longer simulation times. A plot of the first 4 seconds is shown below.

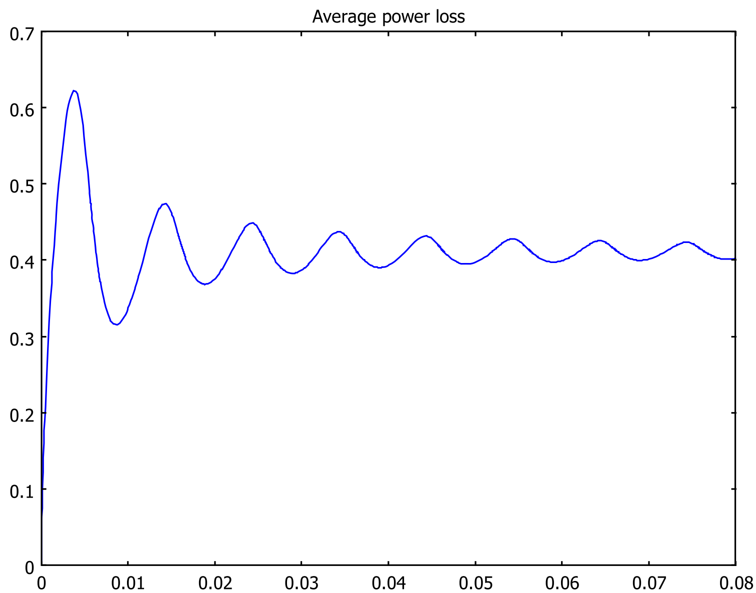

Figure 2: The average power loss dissipated in the magnetic material.

From this figure an average power loss of around 0.4 W can be extracted. The average rms values for the **B** and **H**-fields together with the other extracted values are given in the table below.

EXTRACTED PARAMETER	VALUE
p_{loss}	0.402 W
H_{rms}	1660 A/m
B_{rms}	1.01 T
μ_r'	422
μ_r''	236

References

1. J. P. A. Bastos, N. Sadowski, "*Electromagnetic Modeling by Finite Element Methods*", Marcel Decker 2003, ISBN: 0-8247-4269-9.

2. D. C. Jiles, D. L. Atherton, "*Theory of ferromagnetic hysteresis*", J. Magnetism and Magnetic Materials, Vol. 61, pp. 48-60 (1986)

3. A. Bergqvist, "*A simple vector generalization of the Jiles-Atherton model of hysteresis*", IEEE Trans. Magn., Vol. 32 No.5, pp.4213-5 (1996)

Modeling Using the Graphical User Interface

The steps below are based on opening a model from the AC/DC Model Library.

MODEL NAVIGATOR

1 Click the **Model Library** tab. Browse to the model **AC/DC Module>Electrical Components>amplifier and inductor nocircuit**.

2 Click **OK** and wait for the model to open.

3 Choose **Model Navigator** from the **Multiphysics** menu.

4 Select **2D** from the **Space dimension** list.

5 Select **COMSOL Multiphysics>PDE Modes>PDE, General Form**.

6 Enter `Hr Hz` in the **Dependent variables** edit field, and enter `hyst` in the **Application mode name** edit field.

7 Click the **Add** button.

8 Click **OK** to close the **Model Navigator**.

OPTIONS AND SETTINGS

Constants

1 From the **Options** menu, choose **Constants**.

2 In the **Constants** dialog box, change the existing constant `freq` to `50[Hz]`. Then add the following constants with names and values:

NAME	VALUE	DESCRIPTION
a	560[A/m]	JA parameter
Ms	1.6e6[A/m]	Saturation magnetization
k	1200[A/m]	JA parameter

NAME	VALUE	DESCRIPTION
c	0.1	JA parameter
alpha	0.0007	JA parameter
delta	c/(k*(1-c))*freq^2	Smoothing parameter
tol	1e-3	Tolerance for numerical integration

3 Click **OK**.

Expressions

1 From the **Options** menu, choose **Expressions>Global Expressions**.

2 In the **Global Expressions** dialog box, change the existing expression for `I_coil` to `5[A]*sin(2*pi*freq*t)`. Then add the following variables with names and expressions:

NAME	EXPRESSION	DESCRIPTION
Her	Hr+alpha*Mr_emqa	Effective magnetic field, r-component
Hez	Hz+alpha*Mz_emqa	Effective magnetic field, z-component
normHe	sqrt(Her^2+Hez^2)	Effective magnetic field, norm
Manr	Ms*if(normHe/a<0.1,Her/(3*a),(coth(normHe/a)-a/normHe)*Her/normHe)	Anhysteretic magnetization, r-component
Manz	Ms*if(normHe/a<0.1,Hez/(3*a),(coth(normHe/a)-a/normHe)*Hez/normHe)	Anhysteretic magnetization, z-component
Mrevr	c*(Manr-Mr_emqa)/(1-c)	Reversible magnetization, r-component
Mrevz	c*(Manz-Mz_emqa)/(1-c)	Reversible magnetization, z-component
normMrev	sqrt(Mrevr^2+Mrevz^2)	Reversible magnetization, norm
dMirrrdt	xplus(Mrevr*diff(Her,t)+Mrevz*diff(Hez,t),delta)/(c*k)*if(normMrev>0,Mrevr/normMrev,0)	Time derivative of irreversible magnetization, r-component
dMirrzdt	xplus(Mrevr*diff(Her,t)+Mrevz*diff(Hez,t),delta)/(c*k)*if(normMrev>0,Mrevz/normMrev,0)	Time derivative of irreversible magnetization, z-component
p_loss	Wm/t	Average power loss

3 Click **OK**.

4 From the **Options** menu, choose **Expressions>Scalar Expressions**.

5 In the **Scalar Expressions** dialog box, add the following variables with names and expressions:

NAME	EXPRESSION	DESCRIPTION
`Hr_rms`	`sqrt(quad(at(T,Hr_emqa^2),T,0,t, tol)/t)`	rms value of magnetic field, r-component
`Hz_rms`	`sqrt(quad(at(T,Hz_emqa^2),T,0,t, tol)/t)`	rms value of magnetic field, z-component
`Br_rms`	`sqrt(quad(at(T,Br_emqa^2),T,0,t, tol)/t)`	rms value of magnetic flux density, r-component
`Bz_rms`	`sqrt(quad(at(T,Bz_emqa^2),T,0,t, tol)/t)`	rms value of magnetic flux density, z-component
`mur_biss`	`1/(2*pi*freq*mu0_emqa*avgH2rms)* p_loss/Vcore`	Imaginary part of relative permeability
`mur_prim`	`real(1/mu0_emqa^2*sqrt( avgB2rms/avgH2rms-mur_biss^2))`	Real part of relative permeability

6 Click **OK**.

7 From the **Options** menu, choose **Integration Coupling Variables>Subdomain Variables**.

8 In the **Subdomain Integration Variables** dialog box, add the following variables with names and expressions for the specified subdomain:

NAME	EXPRESSION FOR SUBDOMAIN 2	DESCRIPTION
`dWmdt`	`2*pi*r*(Hr_emqa*diff(Br_emqa, t)+Hz_emqa*diff(Bz_emqa,t))`	Time derivative for the accumulated magnetic energy
`avgH2rms`	`2*pi*r*(Hr_rms^2+Hz_rms^2)/ Vcore`	Average value of the squared rms value of the H-field
`Vcore`	`2*pi*r`	The volume of the core domain
`avgB2rms`	`2*pi*r*(Br_rms^2+Bz_rms^2)/ Vcore`	Average value of the squared rms value of the B-field

9 Click **OK**.

10 From the **Options** menu, choose **Functions**.

11 In the **Functions** dialog box, click the **New** button. In the dialog box that appears, enter `xplus` in the **Function name** edit field. Click **OK**.

12 Enter `x, delta` in the **Arguments** edit field, and `flsmhs(x-delta/2,delta)*x` in the **Expression** edit field.

13 Click **OK**.

PHYSICS SETTINGS

Global Equations

1 From the **Physics** menu, choose **Global Equations**.

2 In the **Global Equations** dialog box, add the following variable with name and equation:

NAME	EQUATION	DESCRIPTION
`Wm`	`Wmt-dWmdt`	Accumulated magnetic energy loss

Subdomain Settings (Azimuthal Currents, emqa)

1 Choose the **Multiphysics** menu and make sure that the **Azimuthal Induction Currents, Vector Potential (emqa)** application mode is active.

2 Go to the **Physics** menu and choose **Subdomain Settings**.

3 In the **Subdomain Settings** dialog box, select subdomain 2, and choose **H = f(B)** from the **Constitutive relation** list.

4 In the **Magnetic field** edit fields, enter `Hr` and `Hz`.

5 Click the **Init** tab, and make sure that the initial condition for all subdomains is set to zero.

6 Click **OK**.

Subdomain Settings (PDE General Form, hyst)

1 Choose the **Multiphysics** menu and choose the **PDE, General Form (hyst)** application mode.

1 Go to the **Physics** menu and choose **Subdomain Settings**.

2 In the **Subdomain Settings** dialog box, select subdomains 1 and 3, and clear the **Active in this subdomain** check box.

3 Select subdomain 2, click the Γ tab, and make sure that all edit fields have zero value.

4 Click the **F** tab and enter the following expressions in the two edit fields

```
diff(Mr_emqa,t)-(1-c)*dMirrrdt-c*diff(Manr,t)
diff(Mz_emqa,t)-(1-c)*dMirrzdt-c*diff(Manz,t)
```

5 Make sure that there are only zero entries in the edit fields under the $\mathbf{e_a}$ and $\mathbf{d_a}$ tabs.

6 Click the **Element** tab and enter `shcurl(2,{'Hr','Hz'})` in the **Shape functions** edit field. This makes the application mode use vector elements of the second order for the in-plane magnetic field.

7 Click **OK**.

Boundary Settings (PDE General Form, hyst)

1 From the **Physics** menu, open the **Boundary Settings** dialog box.

2 Make sure that all active boundaries are set to the Neumann boundary type, and that the edit fields under the **G** tab are all zero.

3 Click **OK**.

MESH PARAMETERS

Use the same mesh as in the original model.

COMPUTING THE SOLUTION

Solver Parameters

1 From the **Solve** menu, choose **Solver Parameters**.

2 In the **Solve Parameters** dialog box, choose **Transient** from the **Analysis** list.

3 Make sure that the **Time dependent** solver is selected, enter `linspace(0,8e-2,201)` in the **Times** edit field, and `1e-3` in the **Relative tolerance** edit field.

4 Click **OK**.

Probe Plot Parameters

1 From the **Postprocessing** menu, choose **Probe Plot Parameters**.

2 Click the **New** button. In the dialog box that appears, use the **Coordinate probe** from the **Plot type** list, and enter `BH` in the **Plot name** edit field.

3 Click **OK** to create the new probe-plot variable.

4 Enter `Hz` in the **Expression** edit field, and leave the coordinate values at the origin.

5 Click the **New** button to create a second probe-plot variable. Choose **Global** from the **Plot type** list, and name the variable `Power loss`. Click **OK**.

6 Choose **Global Expression>Average power loss** from the **Predefined quantities** list. This calculates the average power loss from the accumulated magnetic energy loss.

7 Click **OK** to close the **Probe Plot Parameters** dialog box.

8 Click the **Solve** toolbar button. While solving, you can see the values of the probe plot variables. Increase the size of the **Progress** window if you wish to get larger

plots. When the solution process is finished, the probe plots are copied to two new figures. The plot of the average power loss is shown in Figure 2 on page 6.

POSTPROCESSING AND VISUALIZATION

1 From the **Postprocessing** menu, choose **Plot Parameters**.

2 In the **Plot Parameters** dialog box, click the **Surface** tab. Choose **Magnetization, z-component** from the **Predefined quantities** list.

3 Click the **Contour** tab, select the **Contour plot** check box, and choose **Magnetic potential, phi component** from the **Predefined quantities** list.

4 Click the radio button for **Vector with isolevels**, and enter `-linspace(1e-6,2e-5,10)` in the corresponding edit field.

5 Choose the **cool** color map and click **OK**.

6 Open the **Cross-Section Plot Parameters** dialog box from the **Postprocessing** menu.

7 Click the **Point** tab, and choose **Magnetic flux density, z component** from the **Predefined quantities** list.

8 In the **x-axis data** area, click the radio button for **Expression**, and click the **Expression** button.

9 In the dialog box that appears, choose **Magnetic field, z component** from the **Predefined quantities** list. Click **OK** to close the dialog box.

10 Click **OK** to see the plot in Figure 1 on page 5.